1) [15 points]

In each of the following examples, an electric field **E** is produced that decays as $1/r^n$. Determine the value of n for each case, carefully explaining the reason for your answer.

a) A spherical shell of radius R, centered on the origin, with a surface charge density of $\sigma(\theta) = \sigma_o \cos^3(\theta)$. Here σ_o is a constant and θ is the usual spherical angular coordinate.

b) A disk of radius R in the xy plane at $z = 0$, centered on the origin, with surface charge density $\sigma(r) = \sigma_o \sin(2\pi r/R)$, where r is the radial distance on the disk.

c) Four point charges located as follows:

$+q$ at $\mathbf{r} = (x_o, y_o, z_o)$ $-q$ at $\mathbf{r} = (-x_o, y_o, z_o)$

$+q$ at $\mathbf{r} = (-x_o, -y_o, z_o)$ $-q$ at $\mathbf{r} = (x_o, -y_o, z_o)$

2) [25 points]

a) Consider a dielectric sphere of radius R and real dielectric constant $\varepsilon > 1$, placed in a uniform external electric field $\mathbf{E}_0$. Find the resulting total electric field outside of the sphere.

b) Consider that the uniform external field of part (a) is turned off. A point charge q is now positioned a distance r away from the dielectric sphere. Assume that $R \ll r$. What is the force between the charge and the sphere? Is it attractive or repulsive?

3) [30 points]

Consider a semi-infinite dielectric with a real positive dielectric constant ε, and $\mu=1$. The surface of the dielectric is the xy plane at $z = 0$ and the dielectric fills all of space below this plane. A plane polarized simple harmonic electromagnetic wave of frequency ω is traveling inside the dielectric in the x direction.

a) Write down the boundary conditions that determine how the amplitudes of the electromagnetic fields are related at the interface between the dielectric and the vacuum above it. What do these boundary conditions imply about the relation between the frequencies and the wavevectors of the electromagnetic fields inside the dielectric as compared to outside.

b) Show that the electromagnetic fields decay exponentially as one moves in the z direction away from the surface of the dielectric into the vacuum.

4) [30 points]

For this problem you might need the following formulas:

For a charge distribution with an oscillating electric dipole moment,

$$\mathbf{p}(t) = \mathrm{Re}\left[\mathbf{p}_\omega e^{-i\omega t}\right]$$

the radiated electric and magnetic fields, at position $\mathbf{r}$, in the electric dipole approximation are,

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[-k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{p}_\omega)\right] \quad \mathbf{B}(\mathbf{r},t) = \mathrm{Re}\left[k^2 \frac{e^{i(kr-\omega t)}}{r} \hat{\mathbf{r}} \times \mathbf{p}_\omega\right]$$

where $k = \omega/c$.

Consider now a point charge q, moving in a circular orbit of radius R, centered about the origin in the xy plane. The charge is orbiting counterclockwise with an angular velocity ω.

a) Compute the radiated electric and magnetic fields, expressing your answer in terms of spherical coordinates. Make sure your answers are real valued functions!

b) What is the polarization of the outgoing radiation at a general spherical angle (θ,φ)? What is the polarization when $\theta = 0$? when $\theta = \pi/2$?

c) What is the total radiated energy per one orbit of the charge?