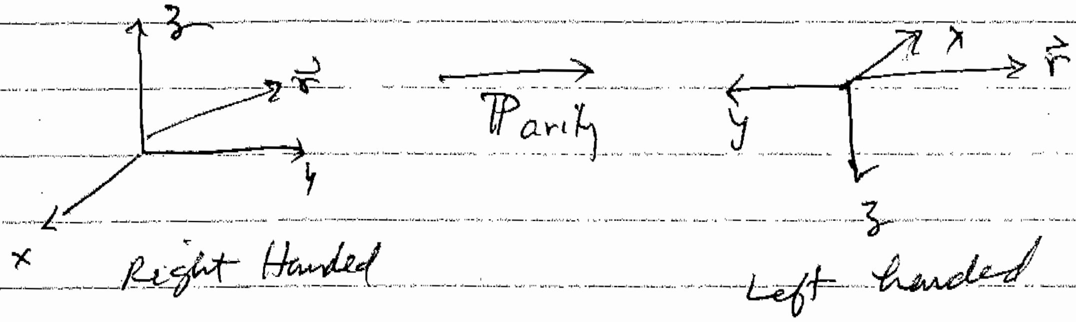
Symmetry under parity transformation

vector vs. psuedo vector



$\vec{r} = (x, y, z) \longrightarrow (-x, -y, -z)$

$\mathbb{P}(\vec{r}) = -\vec{r}$ position $\vec{r}$ is odd under parity

Any vector-like quantity that is odd under $\mathbb{P}$ is a vector.

examples of vectors

position $\vec{r}$

velocity $\vec{v} = \frac{d\vec{r}}{dt}$ since $\vec{r}$ is vector and t is scalar $\mathbb{P}(t) = t$

acceleration $\vec{a} = \frac{d\vec{v}}{dt}$

Force $\vec{F} = m\vec{a}$ since $\vec{a}$ is vector and m is scalar

momentum $\vec{p} = m\vec{v}$ since $\vec{v}$ is vector and m is scalar

electric field $\vec{F} = q\vec{E}$ since $\vec{F}$ is vector and q is scalar $\mathbb{P}(q) = q$

current $\vec{j} = \sum_i q_i \vec{v}_i \, \delta(\vec{r} - \vec{r}_i(t))$

any vector-like quantity that is <u>even</u> under $\mathbb{P}$ is a <u>psuedovector</u>

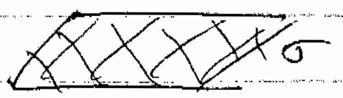
angular momentum $\vec{L} = \vec{r} \times \vec{p}$ since $\vec{r} \to -\vec{r}$ and $\vec{p} \to -\vec{p}$
$\vec{L} \to \vec{L}$ under $\mathbb{P}$
$\vec{L}$ is even under $\mathbb{P}$

magnetic field $\vec{F} = q\,\vec{v} \times \vec{B}$ since $\vec{F}$ and $\vec{v}$ are vectors and q is scalar, $\vec{B}$ must be ~~psudo~~ psuedovector,

cross product of any two vectors is a psuedovector
" " " vector and psuedovector is a vector

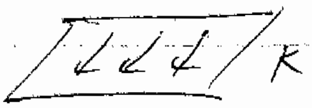
when solving for $\vec{E}$, it can only ~~be~~ ~~b~~ made up of <u>vectors</u> that exist in the problem
When solving for $\vec{B}$, it can only be made up of <u>psuedovectors</u> that exist in the problem

<u>ex</u> charged plane



only directions in problem is normal $\hat{n}$
$\hat{n}$ is a <u>vector</u>
$\vec{E} \propto \hat{n}$

surface current



only directions are the <u>vectors</u> $\hat{n}$ and $\vec{K}$. But $\vec{B}$ can only be made of <u>psuedovectors</u>

$$\Rightarrow \vec{B} \propto (\vec{K} \times \hat{n})$$

Dielectrics + Magnetic Materials - Macroscopic Maxwell Equ

~~Dielectrics~~

Maxwell's equations apply exactly to the true microscopic electric and magnetic fields that arise from all charges and currents

$$\vec{\nabla}\cdot\vec{b} = 0 \qquad \vec{\nabla}\times\vec{e} + \frac{1}{c}\frac{\partial \vec{b}}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{e} = 4\pi\rho_0 \qquad \vec{\nabla}\times\vec{b} = \frac{4\pi}{c}\vec{j}_0 + \frac{1}{c}\frac{\partial\vec{e}}{\partial t}$$

where $\vec{e}$ and $\vec{b}$ are microscopic fields from total charge density ρ_0 and current density $\vec{j}_0$.

However, in most problems involving macroscopic objects, if we took ρ_0 and $\vec{j}_0$ to describe charge + current of each individual atom in a material, then they, and the resulting $\vec{e}$ and $\vec{b}$ would be enormously complicated functions varying rapidly over distances $\sim 10^{-8}$ cm and times $\sim 10^{-16}$ sec.

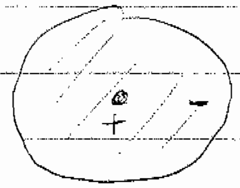
In classical E+M we are generally concerned with phenomena that vary extremely slowly compared to these length + time scales, Rather than worry about the microscopic details of ρ and $\vec{j}$ and resulting $\vec{e}$ and $\vec{b}$ we want to describe phenomena in terms of ~~averaged~~ ~~smoothly varying quan~~ averaged quantities that are smoothly varying at the atomic scale. This results in what are known as the <u>macroscopic</u> <u>Maxwell equations</u>.

<u>Dielectric Materials</u>

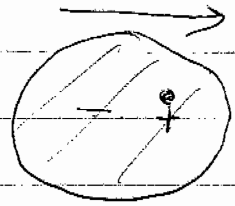
can be solid, liquid or gas

A dielectric material is an insulator. Electrons (valence) are bound to the ionic cores of the atoms. When no electric field is present, the averaged ρ in the dielectric <u>vanishes</u>! One might therefore think that electrodynamics in a dielectric is just due to whatever "extra" or "free" charge is added to the dielectric. However this is not true due to the phenomena of "polarization".

$E = 0$



$E > 0$



electron cloud centered on ionic ~~nucleus~~ core

dipole moment vanishes

electron cloud and ionic core displaced $\vec{d} \propto \vec{E}$

atom is "polarized"

has dipole moment $\vec{p} = q\vec{d} \propto q\vec{E}$

$\vec{p} = \alpha \vec{E}$

atomic polarizability

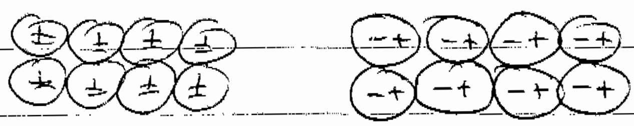
Polarization density $\vec{\mathbb{P}}(\vec{r}) = \sum_i \vec{p}_i \, \delta(\vec{r}-\vec{r}_i)$

$\uparrow$ dipole moment of atom i at position $\vec{r}_i$

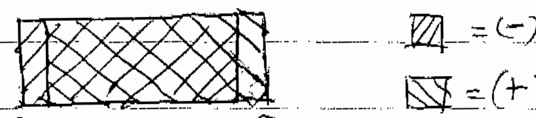
Polarization density $\vec{\mathbb{P}}$ can give rise to regions of net charge – sometimes called "bound charge"

Example

$\vec{\mathbb{P}} \rightarrow$ in terms of averaged charg



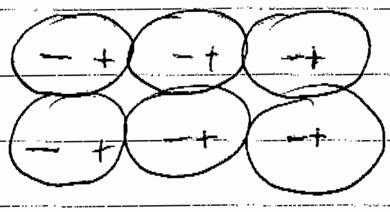
$\vec{E} = 0$ uniform $\vec{E} \rightarrow$ $E > 0$



(−) neutral (+)

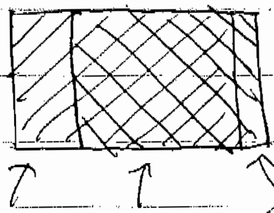
surface charge builds up

For a non uniform $\vec{E}$, atoms are more strongly polarized where E is largest



E strong E weak

$\vec{E} \rightarrow$



(−) $|\rho_+| > |\rho_-|$ (+)

net positive charge region

For uniform $\vec{\mathbb{P}}$, build up surface charge σ_b

For nonuniform $\vec{\mathbb{P}}$, also can build up vol charge density ρ_b.

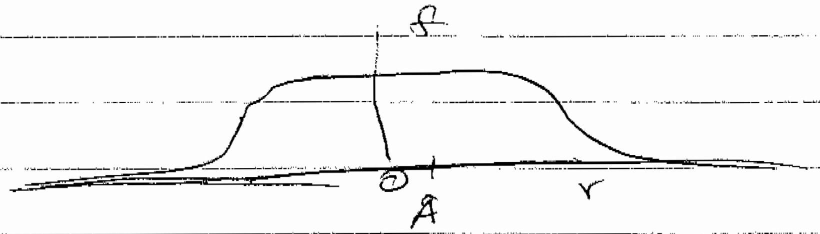
We now carry out the averaging explicitly to see how such polarization enters the macroscopic Maxwell equations

(Jackson 6.6)

Define spatially averaged quantities by

$$\langle F(\vec{r},t)\rangle = \int d^3r'\, f(\vec{r}')\, F(\vec{r}-\vec{r}', t)$$

where $f(\vec{r})$ vanishes for $|\vec{r}|$ large on microscopic length scales, but short on macroscopic length scales. $f(\vec{r})$ normalized to unity $\int d^3r\, f(\vec{r}) = 1$.
Other details of $f(\vec{r})$ are not too important, as long as $f(\vec{r})$ is a smooth function of $\vec{r}$



want $f \approx 1$ for $r < R$
$f \approx 0$ for $r \gg R$
where R is length scal
inbetween micro + macro

$$\frac{\partial}{\partial r_i}\langle F(\vec{r},t)\rangle = \int d^3r'\, f(\vec{r}')\, \frac{\partial F(r-r')}{\partial r_i} = \left\langle \frac{\partial F}{\partial r_i}\right\rangle$$

$$\frac{\partial}{\partial t}\langle F(\vec{r},t)\rangle = \left\langle \frac{\partial F}{\partial t}\right\rangle$$

Define the macroscopic fields

$$\vec{E}(\vec{r},t) \equiv \langle \vec{e}(\vec{r},t)\rangle$$
$$\vec{B}(\vec{r},t) \equiv \langle \vec{b}(\vec{r},t)\rangle$$

Then $\vec{\nabla}\cdot\vec{b} = 0 \Rightarrow \langle \vec{\nabla}\cdot\vec{b} \rangle = 0$

$$\Rightarrow \vec{\nabla}\cdot\langle\vec{b}\rangle = 0$$

$$\Rightarrow \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{e} + \frac{\partial \vec{b}}{\partial t} = 0 \Rightarrow \vec{\nabla}\times\langle\vec{e}\rangle + \frac{\partial}{\partial t}\langle\vec{b}\rangle = 0$$

$$\Rightarrow \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$$

Remaining Maxwell eqn, upon averaging, become

$$\vec{\nabla}\cdot\vec{E} = 4\pi\langle\rho_0\rangle$$
$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\langle\vec{j}_0\rangle + \frac{\partial\vec{E}}{\partial t}$$

Consider $\langle\rho_0\rangle$

$\rho_0 = \sum_i q_i\,\delta(\vec{r}-\vec{r}_i(t))$ sum over all charges

Consider dividing the charge into "free" charges and "bound" charges, where the latter are associated with the molecules that make up the dielectric

$\rho_{free} = \sum_{i\ free} q_i\,\delta(\vec{r}-\vec{r}_i(t))$ sum over only free charges

$$\rho_{bound} = \sum_n \rho_n(\vec{r},t)$$

charge distribution of molecule n

$\rho_n = \sum_{i\in n} q_i\,\delta(\vec{r}-\vec{r}_i(t))$ sum over charges in molecule

$$\langle \rho_n(\vec{r},t)\rangle = \int d^3r'\, f(\vec{r}')\, \rho_n(\vec{r}-\vec{r}',t)$$

$$= \sum_{i\in n} q_i \int d^3r'\, f(\vec{r}')\, \delta(\vec{r} - (\vec{r}' - \vec{r}_i(t)))$$

$$= \sum_{i\in n} q_i\, f(\vec{r} - \vec{r}_i(t))$$

write $\vec{r}_i(t) = \vec{r}_n(t) + \vec{r}_{ni}(t)$

position of center of ~~charge~~ ~~mass~~ mass of molecule n

position of charge i of molecule n with respect to center of ~~mass~~ ~~charge~~ mass

$$\langle \rho_n(\vec{r},t)\rangle = \sum_{i\in n} q_i\, f(\vec{r} - \vec{r}_n - \vec{r}_{ni})$$

Since the $|\vec{r}_{ni}|$ are all of atomic length scale, and f is slowly varying on this length scale, we can expand

$$\langle \rho_n(\vec{r},t)\rangle = \sum_{i\in n} q_i \Big[f(\vec{r}-\vec{r}_n) - (\vec{\nabla} f(\vec{r}-\vec{r}_n))\cdot \vec{r}_{ni} + \frac{1}{2}\sum_{\alpha\beta=1}^{3} \frac{\partial f(\vec{r}-\vec{r}_n)}{\partial r_\alpha \partial r_\beta} (\vec{r}_{ni})_\alpha (\vec{r}_{ni})_\beta + \cdots$$

$$= f(\vec{r}-\vec{r}_n)\Big[\sum_{i\in n} q_i\Big]$$

$$- (\vec{\nabla} f(\vec{r}-\vec{r}_n))\cdot \sum_{i\in n} q_i \vec{r}_{ni}$$

$$+ \sum_{\alpha,\beta=1}^{3} \left(\frac{1}{6} \frac{\partial f(\vec{r}-\vec{r}_n)}{\partial r_\alpha \partial r_\beta}\right) 3\sum_{i\in n} q_i (r_{ni})_\alpha (r_{ni})_\beta$$