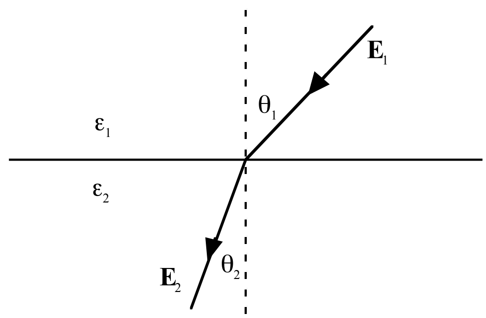
PHY 415 **Final Exam** **Fall 2008**

1) [50 points total] - This problem is a series of relatively short answer questions. The algebra is meant to be relatively simple if you know what you are doing! Each part is worth 10 points. The various parts are unrelated.

a) Consider a charge q located a distance r away from the center of a small neutral dielectric sphere of radius a and dielectric constant ϵ. Assume $a \ll r$. What is the force between the charge and the dielectric sphere? How does it depend on the distance r? Is it attractive or repulsive?

b) Consider two semi-infinite dielectrics with real positive dielectric constants ϵ_1 and ϵ_2 that meet at a plane interface as in the figure below. If a *static* uniform electric field $\mathbf{E}_1$ is present in dielectric 1, making an angle θ_1 with respect to the normal to the interface, then what is the angle θ_2 of the static uniform static electric field $\mathbf{E}_2$ in dielectric 2? Assume that there is no free charge at the interface.



c) Consider a very long straight wire of length L and radius a carrying a uniform steady current I. If the wire has a uniform resistance per unit length, R/L, then there will be a voltage drop down the length of the wire, $V = IR$, and hence an electric field in the wire, $E = V/L$. Find the rate of electromagnetic energy flowing through the surface of the wire. (Assume that L is so long that you may ignore the effects at the ends of the wire.) Does energy flow into or out of the wire? You answer should look familiar. Give a physical explanation for your result.

d) A non-relativistic particle of charge q and initial velocity $\mathbf{v}_0$ hits the surface of a material. As it penetrates the material it loses energy due to multiple scatterings with the atoms of the material until it finally comes to rest. As a result, its velocity at a depth x in the material drops exponentially,

$$\mathbf{v}(x) = \mathbf{v}_0 \mathrm{e}^{-x/s}$$

where s is the mean penetration depth. Compute the total electromagnetic energy radiated until the particle comes to rest.

e) In lecture we saw that the Lorentz tranformation law for electric and magnetic fields was:

$$\begin{aligned} E'_x &= E_x & B'_x &= B_x \\ E'_y &= \gamma(E_y - (v/c)B_z) & B'_y &= \gamma(B_y + (v/c)E_z) \\ E'_z &= \gamma(E_z + (v/c)B_y) & B'_z &= \gamma(B_z - (v/c)E_y) \end{aligned}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the fields in the "laboratory" frame of reference K, and $\mathbf{E}'$ and $\mathbf{B}'$ are the fields in an inertial frame of reference K' that moves with velocity $v\hat{x}$ with respect to K. Here $\gamma = 1/\sqrt{1-(v/c)^2}$.

Consider an infinite line charge oriented along the $\hat{x}$ axis, that in frame K is at rest and holds a charge λ per unit length. Using the Lorentz transformation, compute the electric and magnetic fields observed in the frame K'. Explain these fields in terms of the charge and currents seen in frame K'.

2) [20 points] each part is worth 10 points.

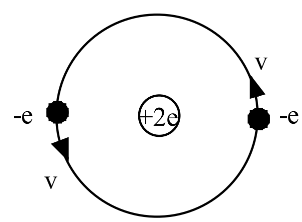
Consider a semi-infinite dielectric with a real positive dielectric constant $\epsilon > 1$ and $\mu = 1$. The surface of the dielectric is the xy plane at $z = 0$ and the dielectric fills all of space below this plane ($z < 0$). A vacuum is above the plane ($z > 0$). A plane polarized simple harmonic electromagnetic wave of frequency ω is traveling inside the dielectric in the $\hat{x}$ direction.

a) Write down the boundary conditions that determine how the amplitudes of the electromagnetic field are related at the interface between the dielectric and the vacuum. What do these boundary conditions imply about the relation between the frequencies and wavevectors of the electromagnetic fields inside and outside the dielectric.

b) Show that the electromagnetic fields decay exponentially as one moves in the $\hat{z}$ direction away from the surface of the dielectric into the vacuum.

3) [30 points total] each part is worth 6 points.

Consider a classical model of the Helium atom as shown below. Two electrons, each with charge $-e$, orbit together on opposite sides of a nucleus of charge $+2e$. The radius of the orbit is a_0 and the electrons orbit with speed v, giving an angular velocity $\omega_0 = v/a_0$.



a) Show that there is no electric dipole radiation in this model.

b) Show that there is no magnetic dipole radiation in this model.

c) Compute the electric quadrapole tensor. What is its frequency of oscillation?

d) If $\mathbf{Q}_\omega$ is the amplitude of the electric quadapole tensor oscillation (i.e. $\mathbf{Q}(t) = \mathrm{Re}\left[\mathbf{Q}_\omega \mathrm{e}^{-i\omega t}\right]$), then the amplitude of the vector potential oscillation for electric quadapole radiation, in the radiation zone approximation, is given by

$$\mathbf{A}_\omega(\mathbf{r}) = -\frac{k^2 \mathrm{e}^{ikr}}{6r} \hat{r} \cdot \mathbf{Q}_\omega$$

What are the amplitudes of the electric field $\mathbf{E}_\omega(\mathbf{r})$ and magnetic field $\mathbf{B}_\omega(\mathbf{r})$ oscillations in this approximation?

e) Using your results from parts (c) and (d), what is the angular distribution of the radiated power $dP/d\Omega$ in this electric quadapole approximation?