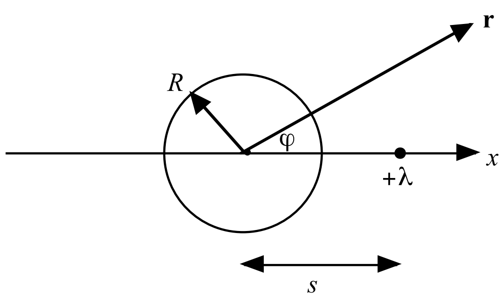
**PHY 415** **Midterm Exam** **Fall 2008**

1) [30 points]
An infinite straight wire oriented in the $\hat{z}$ direction, carrying a uniform line charge $+\lambda$, is placed parallel to an infinite *neutral* (no net charge) conducting cylinder of radius $R$. The wire is a distance $s$ from the center of the cylinder. A cross-sectional picture in the $xy$ plane is shown below.

a) [20 pts] Compute exactly the electrostatic potential $\phi(\mathbf{r})$ at all points $\mathbf{r}$ outside the cylinder.

b) [10 pts] What is the force per unit length on the line charge $+\lambda$? Is it attractive or repulsive? How does this force vary with $s$ for $s \gg R$? How does it vary with $d$ for $s = R + d$, $d \ll R$?




2) [45 points]
A spherical shell of radius $R$ carries a surface charge density $\sigma(\theta) = \sigma_0 \cos^3(\theta)$, where $\theta$ is the usual angle from the $\hat{z}$ axis in spherical coordinates.

a) [20 pts] Compute exactly the electrostatic potential $\phi(\mathbf{r})$ inside and outside the sphere.

b) [20 pts] Explicitly compute the monopole moment $q$, the dipole moment vector $\mathbf{p}$ and the matrix of the quadrapole moment tensor in Cartesian coordinates $Q_{ij}$, for this charge distribution. Use the center of the sphere as the origin.

c) [5 pts] Relate your answers in part (b) to your result of part (a).

3) [25 points]
a) [10 pts] Consider two concentric circular wire loops of radii $a$ and $b$ respectively, centered on the origin in the $xy$ plane at $z = 0$. Assume $b = a + d$ with $d \ll a$. A current $I$ circulates clockwise in the loop of radius $a$, while a current $I$ circulates counterclockwise in the loop of radius $b$. What is the resulting magnetic field to leading order (i.e. to leading power of $1/r$) at an observation point $\mathbf{r}$ far from the loops, $r >> a$?

b) [15 pts] Consider two circular wire loops, both of radius $a$, centered on the origin in the $xy$ plane. One loop is at height $z = +d/2$ and has a current $I$ circulating counterclockwise. The other loop is at height $z = -d/2$ and has a current $I$ circulating clockwise. Assume $d \sim a$. What is the resulting magnetic field to leading order at an observation point $\mathbf{r}$ far from the loops, $r >> a$?

$$P_0(x) = 1 \qquad P_4(x) = \tfrac{1}{8}(35x^4 - 30x^2 + 3)$$

$$P_1(x) = x \qquad P_5(x) = \tfrac{1}{8}(63x^5 - 70x^3 + 15x)$$

$$P_2(x) = \tfrac{1}{2}(3x^2 - 1) \qquad P_6(x) = \tfrac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$$

$$P_3(x) = \tfrac{1}{2}(5x^3 - 3x)$$