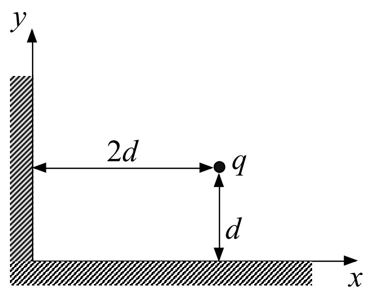
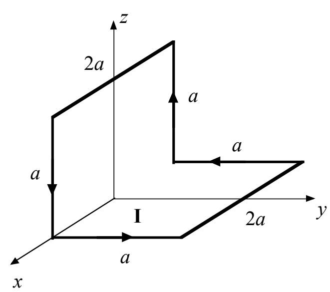
1) [40 points total] - This problem is a series of relatively short answer questions. The algebra is meant to be relatively simple if you know what you are doing! Each part is worth 10 points. The various parts are unrelated.

a) A point charge q is positioned in front of two semi-infinite grounded conducting planes that meet at right angles, as shown in the diagram below. What is the force on the charge q?



b) A wire loop, with rectangular segments in the xy and xz planes, is carrying a steady current I, as shown in the digram below. The lengths of the sides of the loop are a and $2a$ as indicated in the diagram. What is the resulting magnetic field $\mathbf{B}$ at distances $\mathbf{r}$ far from the center of the loop, i.e. $r \gg a$?



c) Consider a very long straight wire of length L and radius a carrying a uniform steady current I. If the wire has a uniform resistance per unit length, R/L, then there will be a voltage drop down the length of the wire, $V = IR$, and hence an electric field in the wire, $E = V/L$. Find the rate of electromagnetic energy flowing through the surface of the wire. (Assume that L is so long that you may ignore the effects at the ends of the wire.) Does energy flow into or out of the wire? You answer should look familiar. Give a physical explanation for your result.

d) In Problem Set 7 you derived the Faraday effect. According to this effect a linearly polarized electromagnetic wave, traveling through a dielectric in the presence of a uniform magnetic field $\mathbf{B}$, has the direction of polarization rotated as it passes through the material. A *naive* application of the principle of superposition might argue against the existence of such an effect: if a linearly polarized wave with fixed direction of polarization, and uniform $\mathbf{B} = 0$, is one solution to Maxwell's equations, and a uniform $\mathbf{B} \neq 0$ and no wave is another solution, then the sum of these two solutions should itself be a solution; hence the presence of the uniform $\mathbf{B} \neq 0$ should have no effect on the propagation of the wave! What is wrong with this naive argument?

2) [20 points]

a) Consider a dielectric sphere of radius R and real dielectric constant $\epsilon > 1$, placed in a uniform external electric field $\mathbf{E}_0$. Find the resulting total electric field outside the sphere.

b) The electric field $\mathbf{E}_0$ of part (a) is turned off. A point charge q is now positioned a distance r away from the dielectric sphere. Assume that $R \ll r$. What is the force between the charge and the sphere? Is it attractive or repulsive?

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3) [20 points]

The electric and magnetic fields of a plane electromagnetic wave, traveling in the $\hat{z}$ axis in a dissipative dielectric media, can be written as:

$$\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \mathrm{Re}\left[\mathbf{E}_\omega \mathrm{e}^{-k_2 z}\mathrm{e}^{i(k_1 z-\omega t)}\right]\\ \mathbf{B}(\mathbf{r},t) &= \mathrm{Re}\left[\frac{c|k|}{\omega}(\hat{z}\times\mathbf{E}_\omega)\mathrm{e}^{-k_2 z}\mathrm{e}^{i(k_1 z-\omega t+\delta)}\right]\end{aligned}$$

where k_1 and k_2 are the real and imaginary parts of the wave vector, $|k| = \sqrt{k_1^2+k_2^2}$, and $\tan\delta = k_2/k_1$.

For a linearly polarizaed wave, where the amplitude $\mathbf{E}_\omega$ is a real vector, $\mathbf{E}$ and $\mathbf{B}$ are orthogonal, i.e. $\mathbf{E}\cdot\mathbf{B} = 0$. However, for a general elliptically polarized wave, this is not longer true!

For an elliptically polarized wave with

$$\mathbf{E}_\omega = E\cos\theta\,\hat{x} + E\sin\theta\,\mathrm{e}^{i\chi}\,\hat{y} \qquad (\theta \text{ and } \chi \text{ arbitrary parameters}) \tag{1}$$

a) Compute the value of $\mathbf{E}\cdot\mathbf{B}$ as a function of space and time. Does $\mathbf{E}\cdot\mathbf{B}$ vary with time or with spatial position?

b) Under what general conditions will $\mathbf{E}\cdot\mathbf{B} = 0$?

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4) [20 points total]

Consider the radiation emitted by a circular wire loop of radius R, centered about the origin in the xy plane at $z = 0$. The current flowing in the loop is given by

$$I(\varphi,t) = \mathrm{Re}\left[I_0\cos(n\varphi)\mathrm{e}^{-i\omega t}\right] \tag{2}$$

where φ is the usual azimuthal angle in spherical coordinates. The frequency ω is such that $R\omega \ll c$.

a) If $n = 0$, show that there is magnetic dipole radiation but no electric dipole radiation.

b) If $n = 1$, show that there is electric dipole radiation but no magnetic dipole radiation.

c) If $n = 2$, show that there is neither electric dipole nor magnetic dipole radiation. What happens in this case? What is the frequency of the emitted radiation? You must explain your answer, not just give a guess.

Legendre Polynomials: $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2-1)$, $P_3(x) = \frac{1}{2}(5x^3-3x)$