Linear Materials

Macroscopic Maxwell Equations

\[ \nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial \vec{B}}{\partial t} = 0 \]

\[ \nabla \cdot \vec{D} = 4\pi \rho \quad \nabla \times \vec{H} = \frac{4\pi}{c} \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \]

where \( \rho \) and \( \vec{j} \) are macroscopic charge and current densities and

\[ \vec{D} = \vec{E} + 4\pi \vec{P} \quad \vec{P} \text{ is polarization density} \]
\[ \vec{H} = \vec{B} - 4\pi \vec{M} \quad \vec{M} \text{ is magnetization density} \]

To close these equations, we will in general need to know how \( \vec{P} \) and \( \vec{M} \) are related to the \( \vec{E} \) and \( \vec{B} \) in the material.

In some materials, there can be a finite \( \vec{P} \) or \( \vec{M} \) even if \( \vec{E} \) or \( \vec{B} \) are zero:

Ferro magnet: \( \vec{M} \) can be non zero even if \( \vec{B} = 0 \)
Ferroelectric: \( \vec{P} \) can be non zero even if \( \vec{E} = 0 \)

But more common are linear materials in which, for small \( \vec{E} \) and \( \vec{B} \), one has \( \vec{P} \propto \vec{E} \) and \( \vec{M} \propto \vec{B} \).
linear dielectric

\[ \vec{P} = \chi_e \vec{E} \quad \text{\( \chi_e \) is "electric susceptibility" \( \chi_e > 0 \) for statics} \]

\[ \vec{D} = \vec{E} + 4\pi \vec{P} = (1 + 4\pi \chi_e) \vec{E} \]
\[ \vec{D} = \varepsilon \vec{E} \quad \text{with} \quad \varepsilon = 1 + 4\pi \chi_e \]

\( \varepsilon \) is the dielectric constant

linear magnetic materials

\[ \vec{M} = \chi_m \vec{H} \quad \text{\( \chi_m \) is "magnetic susceptibility" \( \chi_m > 0 \Rightarrow \) paramagnetic} \]
\[ \chi_m < 0 \Rightarrow \) diamagnetic

\[ \vec{H} = \vec{B} - 4\pi \vec{M} = \vec{B} - 4\pi \chi_m \vec{H} \]
\[ \vec{B} = (1 + 4\pi \chi_m) \vec{H} \]
\[ \vec{B} = \mu \vec{H} \quad \text{with} \quad \mu = 1 + 4\pi \chi_m \]

\( \mu \) is magnetic permeability

For statics, \( \chi_e > 0 \) and \( \chi_m \) (or alternatively \( \varepsilon \) and \( \mu \)) are constants depending on the material.

When we consider dynamics we will see that \( \varepsilon \) becomes a function of frequency.
Clausius-Mossotti equation

Elastic susceptibility & atomic polarizability

If a field $\vec{E}_{oc}$ is applied to an atom, it gets polarized:

$$\vec{P} = \chi \vec{E}_{oc}$$

atomic dipole moment \hspace{1cm} atomic polarizability

$\chi$ is what one calculates from a microscopic theory.

If $\vec{E}_{oc} = \vec{E}$ the average field in the material then electric susceptibility given by:

$$\vec{P} = m \vec{P} = m \chi \vec{E}_{oc} = m \chi \vec{E} = \chi_e \vec{E}$$

$\Rightarrow \chi_e = m \chi$ where $m$ = density of atoms

But a more careful consideration shows $\vec{E}_{oc} \neq \vec{E}$

The average field $\vec{E}$ includes the electric field created by the polarized atom itself, $\vec{E}_{oc}$, the local field the atom sees, should exclude its own self field.

$$\vec{E} = \vec{E}_{oc} + \vec{E}_{atom}$$

average field \hspace{1cm} average field \hspace{1cm} average field of
field excllusive the atom


\[ \vec{E}_{\text{loc}} = \vec{E} - \vec{E}_{\text{atom}} = \vec{E} + \frac{4\pi}{3} \alpha \vec{P} = \vec{E} \frac{2}{3} \alpha \vec{P} \]

\[ \vec{P} = \alpha \vec{E}_{\text{loc}} = \alpha \left( \vec{E} + \frac{4\pi}{3} \alpha \vec{P} \right) = \alpha \vec{E} + \frac{4\pi}{3} \alpha \vec{P} \]

\[ \vec{P} = m \vec{\Phi} = \frac{2m}{1 - \frac{4\pi}{3} m \alpha} \]

\[ \vec{\Phi} = \frac{2}{1 - \frac{4\pi}{3} m \alpha} \]

\[ \kappa_{e} = \frac{m \alpha}{1 - \frac{4\pi}{3} m \alpha} \]
or solve for $\alpha$ in terms of $E$

$$\chi e = \frac{m \chi}{1 - \frac{4\pi}{3} m \chi} \Rightarrow \chi e = \frac{4\pi m \chi}{3} \Rightarrow \alpha = \frac{\chi e}{m(1 + \frac{\chi e}{3})}$$

$$E = 1 + 4\pi \chi e \Rightarrow \alpha = \frac{E - 1}{4\pi m} \left( \frac{1}{1 + \frac{E - 1}{3}} \right)$$

relates atomic polarizability to measured dielectric constant

Clayius Mossohotte
or Lorentz-Lorenz equation

Single model for $\chi$:

$$\chi$$

uniform field inside $E(r) = \frac{4\pi P r}{3}$

atomic radius $a$

$$\rho = \frac{a^3}{\frac{4}{3}\pi a^3}$$

The external field $E_0$, net forces balance $\Rightarrow gE_0 = g \frac{4\pi \rho d}{3}$

$$\chi e = \frac{ma^3}{1 - \frac{4\pi}{3} ma^3}$$

$$p = \frac{g d}{3} \Rightarrow \frac{g E_0}{4\pi} = \frac{3}{4\pi} \frac{(4\pi a^3)}{3} g E_0$$

$$= a^3 E_0 \Rightarrow \alpha = a^3$$

if $f = m \frac{4\pi a^3}{3}$ fraction of void not occupied by atoms

$$\chi e = \frac{1 - \frac{3f}{1 - f}}{4\pi}$$
Linear dielectrics

Bound charge is proportional to free charge

\[ S_b = -\nabla \cdot \vec{P} = -\nabla \cdot \left( \frac{\kappa \varepsilon}{\varepsilon} \vec{E} \right) = -\nabla \cdot \left( \frac{\kappa \varepsilon}{\varepsilon} \vec{D} \right) \]

If \( \kappa \varepsilon \) (and hence \( \varepsilon \)) is spatially constant, then

\[ S_b = -\frac{\kappa \varepsilon}{\varepsilon} \nabla \cdot \vec{D} = -\frac{\kappa \varepsilon}{\varepsilon} 4\pi \rho \]

\[ S_b = -\frac{4\pi \kappa \varepsilon}{1 + 4\pi \kappa \varepsilon} \rho \]

when free charge \( \rho = 0 \),

then \( S_b = 0 \)

\[ S_{\text{total}} = S + S_b = \rho \left[ 1 - \frac{4\pi \kappa \varepsilon}{1 + 4\pi \kappa \varepsilon} \right] = \frac{\rho}{1 + 4\pi \kappa \varepsilon} = \frac{\rho}{\varepsilon} = S_{\text{total}} \]

Bound charge "screens" the free charge so the total charge is reduced compared to the free charge.
For linear dielectrics

**Statics**

\[ \nabla \cdot \mathbf{D} = \varepsilon \varepsilon_0 \rho \]
\[ \nabla \times \mathbf{E} = \mathbf{0} \]

\[ \mathbf{D} = \varepsilon \mathbf{E} \Rightarrow \nabla \cdot (\varepsilon \mathbf{E}) = \varepsilon \varepsilon_0 \rho \]

If \( \varepsilon \) is constant in space then

\[ \varepsilon \nabla \cdot \mathbf{E} = \varepsilon \varepsilon_0 \rho \]
\[ \nabla \times \mathbf{E} = \mathbf{0} \]

\( \varepsilon \) looks just like ordinary electrostatics but with \( \rho \Rightarrow \frac{\rho}{\varepsilon} \)

Alternatively, could write \( \mathbf{E} = \frac{\mathbf{D}}{\varepsilon} \)

\[ \Rightarrow \nabla \times \left( \frac{\mathbf{D}}{\varepsilon} \right) = \mathbf{0} \]
\[ \Rightarrow \nabla \times \mathbf{D} = \mathbf{0} \] when \( \varepsilon \) constant in space

\[ \nabla \cdot \mathbf{D} = \varepsilon \varepsilon_0 \rho \] looks just like ordinary electrostatics, but with \( \mathbf{E} \Rightarrow \mathbf{D} \)

Complication arises at interface between dielectrics (or between dielectric and vacuum). At interface, \( \varepsilon \) is not constant \( \Rightarrow \nabla \times \mathbf{D} \neq \mathbf{0} \).

What we can do is to solve for \( \mathbf{E} \) or \( \mathbf{D} \) inside each dielectric separately, and then use the boundary conditions

\[ \hat{n} \cdot (\mathbf{D}_{\text{above}} - \mathbf{D}_{\text{below}}) = \varepsilon \varepsilon_0 \]
\[ \hat{n} \cdot (\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}}) = 0 \]

to match solutions across the interfaces.

A similar story holds for linear magnetic materials.
Single example: parallel plate capacitor filled with a dielectric

\[ \sigma \]

\[ \bar{D} = \bar{D} = 0 \] outside plates

Between plates \( \bar{D} = \bar{D} = 0 \) as \( \rho = 0 \)

\[ \nabla \cdot \bar{D} = 0 \Rightarrow \bar{D} = 0 \Rightarrow \bar{D} \text{ is constant} \]

Boundary conditions:

Left side plate

\[ \hat{m} = \hat{x} \]

\[ D = 0 \]

\[ \hat{x} \cdot (\bar{D} \text{ above } - \bar{D} \text{ below}) = D = 4\pi \sigma \]

Right side plate

\[ \hat{m} = \hat{x} \]

\[ D = 0 \]

\[ \hat{x} \cdot (\bar{D} \text{ above } - \bar{D} \text{ below}) = -D = 4\pi (-\sigma) \]

\[ D = 4\pi \sigma \text{ as before} \]

\[ \Rightarrow \bar{D} = 4\pi \sigma \hat{x} \]

\[ \bar{E} = \frac{\bar{D}}{\varepsilon} = \frac{4\pi \sigma}{\varepsilon} \hat{x} \]

Electric field reduced by factor \( \frac{\varepsilon}{\varepsilon_0} \) as compared to capacitor with vacuum between plates.

See Jackson section 4.4 for more interesting examples.

Dielectric sphere in uniform applied \( \varepsilon \)

See Jackson section (5.12) for an interesting magnetic h.c. problem.
point charge within a dielectric sphere

\[ \Phi \text{ charge } q \text{ at center of dielectric sphere of radius } R, \text{ dielectric const } \varepsilon \]

\[ \vec{V} \cdot \vec{D} = 4\pi q = \oint_S \hat{n} \cdot \vec{D} = 4\pi q \text{ and} \]

From symmetry \( \vec{D}(r) = \vec{D}(r) \hat{r} \)

\[ \oint_S \hat{n} \cdot \vec{D} = 4\pi R^2 \vec{D}(r) = 4\pi q \]

Sphere of radius \( r \)

\[ \vec{D} = \frac{\vec{E}}{r^2} \hat{r} \quad \text{all } r \]

\[ \Rightarrow \vec{E}(r) = \begin{cases} \frac{\varepsilon}{r^2} \hat{r} & r < R \\ \frac{q}{r^2} \hat{r} & r > R \end{cases} \]

can check that tangential component of \( \vec{E} \) is continuous and normal component of \( \vec{D} \) is continuous as there is no free \( \sigma \) at surface of dielectric.

normal component of \( \vec{E} \) jumps by

\[ \vec{n} \cdot (\vec{E}_{\text{above}} - \vec{E}_{\text{below}}) = \frac{\varepsilon}{R^2} - \frac{\varepsilon}{ER^2} = \frac{\varepsilon}{R^2} \left( 1 - \frac{1}{E} \right) = \frac{\varepsilon}{R^2} \left( \frac{E-1}{E} \right) \]

\[ = \frac{\varepsilon}{R^2} \left( \frac{4\pi \varepsilon_0}{1 + 4\pi \varepsilon_0} \right) = \frac{4\pi \sigma_{\text{total}}}{R^2} = \frac{4\pi \sigma_b}{R^2} \]

\[ \Rightarrow \sigma_b = \frac{\varepsilon}{4\pi R^2} \left( \frac{4\pi \varepsilon_0}{1 + 4\pi \varepsilon_0} \right) = \frac{\varepsilon \varepsilon_0}{R^2 \varepsilon} \]

We can check this directly.
\[
\vec{P} = \frac{\kappa e \vec{E}}{\varepsilon} = \frac{\kappa e E}{r^2} \hat{r}
\]

\[
\vec{P} = -\nabla \cdot \vec{P} = -\frac{\kappa e g}{\varepsilon} \cdot 4\pi \delta(r)
\]

bound charge at origin: \( g_b = -\frac{\kappa e}{\varepsilon} \cdot 4\pi \delta \)

total charge at origin: \( g + g_b = g \left(1 - \frac{4\pi \kappa e}{\varepsilon}\right) \)

\[
\varepsilon = 1 + 4\pi \kappa e = \frac{g}{g} \left(\frac{\varepsilon - 4\pi \kappa e}{\varepsilon}\right) = \frac{\varepsilon}{\varepsilon} \text{ screened charge}
\]

at surface,

\[
\vec{E} = \hat{m} \cdot \vec{P} = \frac{\kappa e \vec{E}}{\varepsilon} \frac{g}{R^2} \text{ agrees with what we get from jump in } \hat{m} \cdot \vec{E}
\]

Note: inside the dielectric the \( \vec{E} \) field is that of the screened point charge \( \frac{g}{\varepsilon} \), outside the dielectric \( \vec{E} \) is just that of the free charge \( g \). There is no evidence in \( \vec{E} \) out that the dielectric even exists!
Now consider same problem but $q$ is off center.

\[ \nabla \cdot \vec{D} = 4\pi \rho \quad \text{where} \quad \rho = \frac{q}{\epsilon} \delta(r^2 - s^2) \]

\[ \vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = \frac{4\pi \rho}{\epsilon} \]

\[ \vec{E} = -\nabla \phi \Rightarrow \nabla^2 \phi = -\frac{4\pi \rho}{\epsilon} = -\frac{4\pi q}{\epsilon} \delta(r^2 - s^2) \]

Solution for $\phi$ will be of the form

\[ \phi(r) = \frac{q}{\epsilon \sqrt{r^2 - s^2}} + F(r) \]

where 1st term is due to the point charge $q/\epsilon$ and 2nd term satisfies $\nabla^2 F = 0$ and will be chosen to get the correct behavior at the boundary of the dielectric.

Since there is azimuthal symmetry about $\hat{z}$, we can write

\[ F(r) = \sum_{l=0}^{\infty} a_l r^l p_l(\cos \theta) \]

there are no be terms since $F$ should not diverge at the radial origin.
So inside $r < R$

$$\phi^i(r) = \frac{q}{\sqrt{r-s^2}} + \sum_{\ell=0}^{\infty} a_{\ell} r^{\ell+1} P_\ell(\cos \theta)$$

From our discussion of electric multipole expansion, we know we can write for $r > s$,

$$\frac{1}{\sqrt{r-s^2}} = \frac{1}{r} \sum_{\ell=0}^{\infty} \frac{(s/r)^\ell}{\ell!} P_\ell(\cos \theta)$$

So for $r > s$ (not true for $r < s$ !)

$$\phi^i(r) = \sum_{\ell=0}^{\infty} \left( \frac{q}{\ell!} \left( \frac{s}{r} \right)^\ell + a_{\ell} r^{\ell+1} \right) P_\ell(\cos \theta)$$

Outside the sphere there is no charge, so $\nabla \cdot \vec{E} = 0$ or $\nabla^2 \phi = 0$

$$\Rightarrow \phi^{out}(r) = \sum_{\ell=0}^{\infty} \frac{b_{\ell}}{r^{\ell+1}} P_\ell(\cos \theta)$$

There are no $a_{\ell} r^{\ell+1}$ terms since $\phi^{out} \to 0$ as $r \to \infty$

To determine the unknown $a_\ell$ and $b_\ell$ we use the boundary conditions at surface of dielectric at $r = R$
1. Tangential component \( \vec{E} \) is continuous

\[
\vec{E} = -\frac{\partial \phi}{\partial r} \hat{r} - \frac{1}{r} \frac{\partial \phi}{\partial \theta} \hat{\theta} = \vec{E}_r \hat{r} + \vec{E}_\theta \hat{\theta}
\]

\( \Rightarrow \) \( \vec{E}_\theta \) is continuous at \( r = R \)

condition that \( \vec{E}_\theta \) is continuous is the same
condition that \( \phi \) is continuous (check this out
for yourself if you are not sure)

\( \Rightarrow \) \( \phi^\text{in} (R,\theta) = \phi^\text{out} (R,\theta) \)

\[
\frac{q}{\varepsilon R} \left( \frac{S}{R} \right)^l + a_2 R^l = \frac{b e}{R^{l+1}}
\]

\( \Rightarrow \)

\[
b e = \frac{q}{\varepsilon} \frac{S^l}{l} + a_2 R^{2l+1}
\]

Normal component \( \vec{D} \) is continuous (since free surface charge \( \sigma = 0 \))

\[
\vec{D} = \varepsilon \vec{E}
\]

\( \Rightarrow \) \( \varepsilon \vec{E}^\text{in} = \varepsilon \vec{E}^\text{out} \)

\[
-\varepsilon \frac{\partial \phi^\text{in}}{\partial r} \bigg|_R = -\varepsilon \frac{\partial \phi^\text{out}}{\partial r} \bigg|_R
\]

\( \Rightarrow \)

\[
\frac{(l+1) q}{R^2} \left( \frac{S}{R} \right)^l - l e a_2 R^{l-1} = \frac{(l+1) b e}{R^{l+2}}
\]
\[ q s^l - \frac{b}{\varepsilon} e a e R^{2l+1} = b e \]

Substitute \( b e \) from previous boundary condition

\[ q s^l - \frac{b}{\varepsilon} e a e R^{2l+1} = \frac{q}{\varepsilon} s^l + a e R^{2l+1} \]

\[ q s^l \left[ 1 - \frac{1}{\varepsilon} \right] = a e R^{2l+1} \left[ 1 + \frac{b}{\varepsilon} e \right] \]

\[
\begin{align*}
    a_l &= \frac{q s^l}{R^{2l+1}} \frac{\left[ 1 - \frac{1}{\varepsilon} \right]}{\left[ 1 + \left( \frac{b}{\varepsilon} e \right) \right]} \\
    b_l &= \frac{q}{\varepsilon} s^l + a e R^{2l+1}
\end{align*}
\]

\[ = \frac{q}{\varepsilon} s^l + q s^l \frac{\left[ 1 - \frac{1}{\varepsilon} \right]}{\left[ 1 + \left( \frac{b}{\varepsilon} e \right) \right]} \]

\[ b_l = \frac{q s^l}{\varepsilon} \left[ 1 + \frac{\varepsilon - 1}{1 + \left( \frac{b}{\varepsilon} e \right)} \right] \]

\[ = \frac{q s^l}{\varepsilon} \left[ \frac{\varepsilon (1 + \frac{b}{\varepsilon} e)}{1 + \left( \frac{b}{\varepsilon} e \right) \varepsilon} \right] \]

\[
\begin{align*}
    b_l &= \frac{q s^l}{\varepsilon} \left[ 1 + \left( \frac{b}{\varepsilon} e \right) \right] \left[ 1 + \left( \frac{b}{\varepsilon} e \right) \varepsilon \right]
\end{align*}
\]
check the result:

as \( s \to 0 \), should recover previous answer

for \( s = 0 \), \( a_e = b_e = 0 \) for all \( l \neq 0 \)

\[
a_0 = \frac{q}{R} \left[ 1 - \frac{1}{e} \right]
\]

\[
b_0 = \frac{q}{e}
\]

\[
S_0 \quad \hat{\Phi}^\nu(r) = \frac{q}{er} + \frac{q}{R} \left[ 1 - \frac{1}{e} \right]
\]

\[
\tilde{E}^\nu = -\nabla \hat{\Phi}^\nu = \frac{q}{er^2} \hat{r} \quad \text{as before}
\]

\[
\hat{\Phi}^{\text{out}}(r) = \frac{q}{r}
\]

\[
\hat{E}^{\text{out}} = -\nabla \hat{\Phi}^{\text{out}} = \frac{q}{r^2} \hat{r} \quad \text{as before,}
\]

\( \hat{r} \) is the constant that is the 2nd term in \( \hat{\Phi}^\nu \)

is just what is needed to make \( \hat{\Phi} \) continuous at \( r = R \).
another check:

let $\epsilon \to \infty$ this models a conductor

again one finds $a_e = b_e = 0$ for all $\epsilon > 0$

$$a_0 = \frac{q}{R}$$

$$b_0 = q$$

$$\phi_{in}(\vec{r}) = \sum_{\epsilon \in r(\vec{r})} \frac{\epsilon}{\epsilon R} e^{\frac{R}{\epsilon \vec{r}}} \to \frac{q}{R} \quad \text{as} \quad \epsilon \to \infty$$

$$\Rightarrow E_{in}(\vec{r}) = 0 \quad \text{as} \quad \phi_{in} \text{ is a constant.}$$

$$\phi_{out}(\vec{r}) = \frac{q}{R} \quad \Rightarrow \quad E_{out} = \frac{q}{r^2} \hat{r}$$

field outside is like point charge $q$ at the origin, independent of where $q$ is inside the sphere.
This is the correct behavior of a conductor.

The mobile charges in the conductor completely screen the $q$ inside, and leave a uniform surface charge $\sigma_f = \frac{q}{4\pi R^2}$ on the surface.
Magnetic states

Bar magnets \( \mathbf{f} = 0 \), \( \mathbf{H} \) fixed and given (not a linear material)

\[
\nabla \cdot \mathbf{B} = 0
\]

\[
\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} = 0
\]

\[
\nabla \times \mathbf{H} = 0 \Rightarrow \mathbf{H} = -\nabla \Phi_m \quad \text{magnetic scalar potential}
\]

\[
\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}
\]

\[
\nabla \cdot \mathbf{B} = \nabla \cdot (\mathbf{H} + 4\pi \mathbf{M}) = 0
\]

\[
\nabla \cdot \mathbf{H} = -\nabla^2 \Phi_m = -4\pi \nabla \cdot \mathbf{M}
\]

\[
\nabla^2 \Phi_m = 4\pi \nabla \cdot \mathbf{M}
\]

so \( \Phi_m \equiv -\nabla \cdot \mathbf{H} \) looks like a magnetic "charge"

\( \Phi_m \) is source for \( \mathbf{H} \)

Also at surfaces of material \( \Sigma_m = \mathbf{M} \cdot \mathbf{H} \) looks like surface charge

\[
\mathbf{H}(\mathbf{r}) = \int d^3 \mathbf{r}' \, \Phi_m(\mathbf{r}') \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} + \oint da' \, \mathbf{M}(\mathbf{r}') \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}
\]

Field lines for \( \mathbf{H} \) can start and end at sources and sinks given by \( \Phi_m \) and \( \Sigma_m \)
\[ \vec{M} = M \hat{z} \]

Bound currents

\[ \oint \vec{J} = c \vec{\nabla} \times \vec{M} = 0 \]

\[ \vec{K}_b = c \vec{M} \times \hat{z} \]

\[ \vec{K}_b = \{ \begin{array}{ll} c \vec{M} \Phi & \text{on side} \\ 0 & \text{on top and bottom} \end{array} \]

\( \vec{K}_b \) is like solenoid current.

Field lines of \( \vec{B} \) look like:

Field lines of \( \vec{H} \) look like parallel plate capacitor.

Field lines of \( \vec{H} \) = field lines of \( \vec{B} \) outside magnet, but they are very different inside the magnet!

But \( \vec{H} \) is determined as follows:

\[ \vec{B}_M = -\vec{\nabla} \times \vec{M} = 0 \]

\[ \vec{H}_M = \vec{M} \times \hat{z} = \{ M \text{ on top} \\
- M \text{ on bottom} \} \]

field lines of \( \vec{H} \) look like parallel plate capacitor.