PHY 415 **Midterm Exam** **Fall 2010**

1) [30 points]

A solid conducting sphere of radius R is connected to a battery which keeps it at a constant potential ϕ_0, relative to a reference point at infinity (i.e. $\phi \to 0$ as $r \to \infty$). Assume $\phi_0 > 0$.

a) What is the total amount of charge that the battery must deposit on the conducting sphere to keep it at potential ϕ_0? [5 pts]

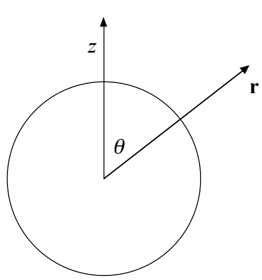
b) A point charge q ($q > 0$) is placed a distance r from the center of the sphere. Now what is the total amount of charge on the conducting sphere? [10 pts]

c) What is the force between q and the conducting sphere? Is it attractive or repulsive (you must say *why*)? [8 pts]

d) Suppose that a cavity exists in the interior of the conducting sphere, and a charge Q is inside the cavity. Now what is the force on the charge q outside? [7 pts]

2) [40 points]

Consider a hollow spherical shell of radius R, with a fixed electrostatic potential on its surface given by $\phi(R, \theta) = \phi_0 \cos^2 \theta$. Here θ is the usual polar angle in spherical coordinates, and the reference point is taken so that $\phi \to 0$ as $r \to \infty$.



a) Find the electrostatic potential $\phi(\mathbf{r})$ inside the shell. [10 pts]

b) Find the electrostatic potential $\phi(\mathbf{r})$ outside the shell. [10 pts]

c) What is the surface charge density $\sigma(\theta)$ on the surface of the sphere? [10 pts]

d) What is the total amount of charge on the sphere? If we put the origin at the center of the sphere, does this charge distribution have an electric dipole moment? an electric quadrupole moment? (you must explain the reason for your answer to get credit) [10 pts]

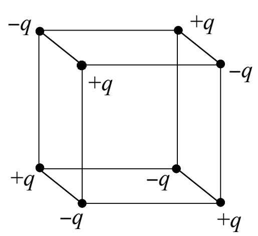
3) [30 points]

In each of the following situations, the electric field $\mathbf{E}$ or magnetic field $\mathbf{B}$ will decay with the distance r from the source as some power of the distance, i.e. as $1/r^n$. For each situation, give the value of n and briefly explain why.

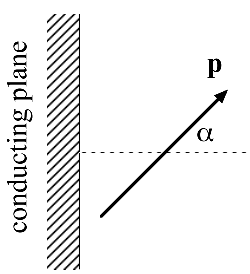
a) A spherical shell of radius R, is centered on the origin, with a surface charge density of $\sigma(\theta) = \sigma_0 \cos^3(\theta)$. Here σ_0 is a constant, and θ is the usual spherical polar angular. [6 pts]

b) A disk of radius R in the xy plane at $z = 0$, is centered on the origin, with surface charge density $\sigma(r) = C \sin(2\pi r/R)$. Here r is the radial distance from the center of the disk. [6 pts]

c) Eight point charges are located at the corners of a cube with side of length a. The charges are all of equal magnitude, but alternate in sign as one goes around any of the faces of the cube. [6 pts]



d) An electric dipole $\mathbf{p}$ is oriented at an angle of α with respect to an infinite flat conducting plane, as shown. We are interested in the electric field far from the dipole, in front of the conducting plane. What is the value of n when $\alpha = 0°$, $45°$, $90°$? [6 pts]



e) Two circular wire loops, each of radius R, are centered about the origin in the xy plane, one at height $z = +d/2$, the other at height $z = -d/2$. The first has current I circulating counterclockwise, the second has current I circulating clockwise. Assume d and R are of comparable length. [6 pts]

Legendre Polynomials:

$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{1}{2}(3x^2 - 1),\ P_3(x) = \frac{1}{2}(5x^3 - 3x)$