1) [30 points total]

a) [10 points] Write down the microscopic Maxwell's equations. Describe (in words) a physical phenomenon that is associated with each term.

b) [10 points] When we left statics for dynamics, we saw that the "dielectric constant" ϵ was really a complex function of frequency $\epsilon(\omega)$. Describe two consequences of the fact that ϵ varies with ω. Describe two consequences of the fact that ϵ may be complex.

c) [10 points] Explain what is meant by a gauge transformation in electrodynamics.

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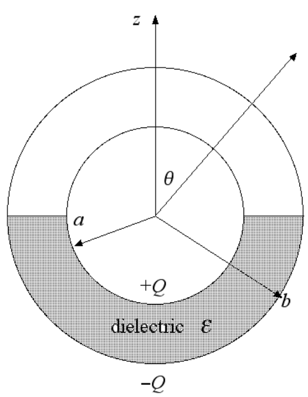
2) [25 points total]

Two thin concentric conducting spherical shells of inner radius a and outer radius b carry total free charges $+Q$ and $-Q$ respectively. The space between them is half filled by a dielectric material with dielectric constant ϵ, as sketched below.



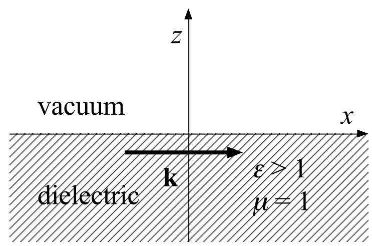
a) Find the electric field $\mathbf{E}$ everywhere between the shells, $a < r < b$.

b) Calculate the total surface charge density $\sigma_{\text{tot}}(\theta)$ on the surface of the inner shell at $r = a$.

c) Calculate the induced bound surface charge density $\sigma_b(\theta)$ on the surface of the dielectric at $r = a$.

3) [20 points total]

Consider a semi-infinite dielectric with a real positive dielectric constant $\epsilon > 1$ and $\mu = 1$. The surface of the dielectric is the xy plane at $z = 0$ and the dielectric fills all of space below this plane ($z < 0$). A vacuum is above the plane ($z > 0$). A plane polarized simple harmonic electromagnetic wave of frequency ω is traveling inside the dielectric in the $\hat{\mathbf{x}}$ direction, as sketched below.



a) [10 points] Write down the boundary conditions that determine how the amplitudes of the electromagnetic field are related at the interface between the dielectric and the vacuum. What do these boundary conditions imply about the relation between the frequencies and wavevectors of the electromagnetic fields inside and outside the dielectric.

b) [10 points] Show that the electromagnetic fields decay exponentially as one moves in the $\hat{\mathbf{z}}$ direction away from the surface of the dielectric into the vacuum.

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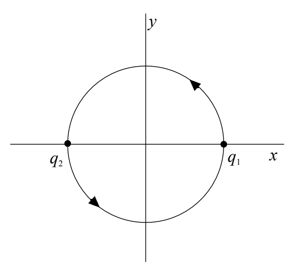
4) [25 points total]

Consider two charges, q_1 and q_2, which are moving in a circular orbit of radius d with angular frequency ω in the xy plane, as sketched below.



a) [15 points] If $q_1 = -q_2 \equiv q$, find the radiated $\mathbf{E}$ and $\mathbf{B}$ fields in the electric dipole approximation. Express your answers as REAL functions of space and time. Find the time averaged Poynting vector as a function of space. Make a polar plot of the angular distribution of the radiated power, $dP/d\Omega$.

b) [10 points] What happens if one now has the case $q_1 = q_2 \equiv q$? What term in the multipole expansion is responsible for the radiation? What is the frequency of the radiation? You do not need to explicitly calculate $\mathbf{E}$, $\mathbf{B}$ or $\mathbf{S}$, but you must explain your reasoning clearly.

Hint: For a time dependent charge distribution with an oscillating electric dipole moment given by $\mathbf{p}(t) = \mathrm{Re}[\mathbf{p}_\omega e^{-i\omega t)}]$, the radiated fields in the electric dipole approximation are given by,

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[-k^2\frac{e^{i(kr-\omega t)}}{r}\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{p}_\omega)\right] \quad , \quad \mathbf{B}(\mathbf{r},t) = \mathrm{Re}\left[k^2\frac{e^{i(kr-\omega t)}}{r}\hat{\mathbf{r}}\times\mathbf{p}_\omega\right] \quad \text{, where } k = \omega/c$$