PHY 415 **Midterm Exam** **Fall 2011**

1) 40 points total

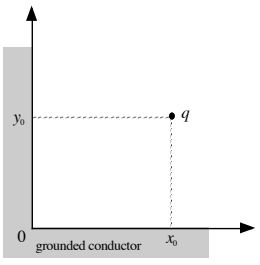
This problem is a set of four short answer questions. The calculation needed for each part should be brief.

a) [10 pts] Explain why Ampere's Law as first found in magnetostatics, $\nabla \times \mathbf{B} = (4\pi/c)\mathbf{j}$, cannot in general remain correct when one considers dynamic situations with moving charges. Give a very brief mathematical argument.

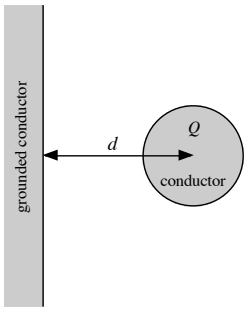
b) [10 pts] Consider a point charge q in front of two infinite grounded planes at right angles to each other, as shown in the figure below. The intersection of the planes coincides with the $\hat{z}$ axis, and the position of the charge is given by the coordinates $\mathbf{r} = (x_0, y_0, 0)$. What is the force $\mathbf{F}$ on the charge q?

c) [10 pts] Consider a solid conducting sphere of radius R containing total charge Q. The sphere is placed a distance d in front of an infinite grounded conducting plane. To leading order, what is the force on the sphere when it is far from the plane, $d \gg R$? The sphere is now positioned close to the plane. Is this problem easily solved by the image charge method? If yes, then explain what are the needed image charges. If not, then explain why not.

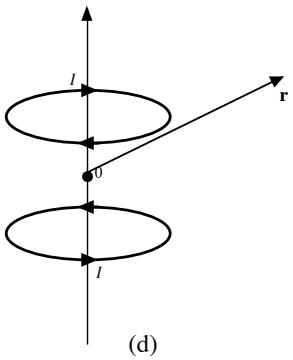
d) [10 pts] Two circular wire loops, each of radius R, are centered about the origin and lying in the xy plane. One loop is at height $z = +a$ and has current I circulating clockwise. The other loop is at height $z = -a$ and has a current I circulating counter-clockwise. For large distances $r \gg R$ the amplitude of the magnetic field B will decay, to leading order, proportional to $B \sim 1/r^n$. What will be the value of n? You must explain your answer.



(b)



(c)



(d)

2) 30 points total

Consider a sphere of radius R, whose surface is fixed to the potential $\phi(R, \theta, \varphi) = \phi_0 \cos^3 \theta$, where θ and φ are the usual spherical angles.

a) [10 pts] What is the electrostatic potential ϕ inside the sphere?

b) [10 pts] What is the electrostatic potential ϕ outside the sphere?

c) [10 pts] What is the surface charge density σ on the surface of the sphere?

Legendre Polynomials:

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$, $P_3(x) = \frac{1}{2}(5x^3 - 3x)$

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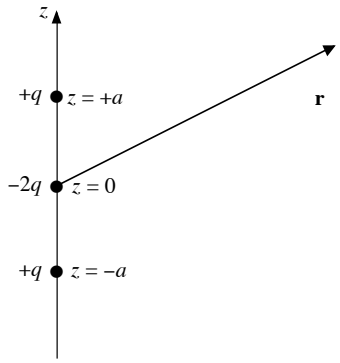
3) 30 points total

Consider three charges arranged in a line along the $\hat{z}$ axis, as in the diagram below:



Find the electrostatic potential $\phi(\mathbf{r})$ at the observation point $\mathbf{r}$ to leading order in $1/r$, using the multipole expansion, with the origin fixed as shown. Express your answer in spherical coordinates, $\phi(r, \theta, \varphi)$.