PHY 415 **Final Exam** **Fall 2013**

1) [35 points total]

a) [20 pts] Consider a dielectric sphere of radius R and real dielectric constant $\epsilon > 1$, placed in a uniform external electric field $\mathbf{E}_0 = E\hat{z}$. Find the resulting total electric field $\mathbf{E}(\mathbf{r})$ outside the sphere.

b) [15 pts] Consider that the uniform external field of part (a) is turned off. A point charge q is now positioned a distance r away from the dielectric sphere. Assume that $R \ll r$. What is the force between the charge and the sphere? Is it attractive or repulsive?

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2) [35 points total] Consider an isolated neutral atom where the nucleus of the atom is held fixed in place at the origin of some coordinate system. A plane, linearly polarized, electromagnetic wave of angular frequency ω impinges on the atom, exerting a force on the electron cloud and causing it to oscillate. If $\mathbf{r}(t)$ denotes the center of mass of the electron cloud with respect to the nucleus, the electron cloud moves under the influence of three forces: (i) the force from the electromagnetic wave, (ii) a restoring force $-m\omega_0^2\mathbf{r}$ pulling the electron back to the nucleus, and (iii) a dissipative force $-m\gamma\mathbf{v}$. Here m is the mass of the electron cloud, $-q$ its charge, and $\mathbf{v} = \dot{\mathbf{r}}$ the velocity of the electron cloud's center of mass. You may approximate the electron cloud as a fixed rigid object moving in a classical non-relativistic way.

a) [20 pts] What is the total work done on the electron cloud by the electromagnetic wave in one period of osciallation?

b) [10 pts] What is the total energy radiated away from the atom in electromagnetic radiation in one period of oscillation?

c) [5 pts] If one wishes to interpret the dissipative force $-m\gamma\mathbf{v}$ as arising solely from the radiated energy, what should be the value of the parameter γ? Do you see anything funny about your answer? Is there a physical interpretation?

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3) [30 points total]

Consider the radiation emitted by a circular wire loop of radius R, centered about the origin in the xy plane at $z = 0$. The current flowing in the loop is given by

$$I(\varphi, t) = \mathrm{Re}\left[I_0 \cos(n\varphi)\mathrm{e}^{-i\omega t}\right]$$

where φ is the usual azimuthal angle in spherical coordinates. The frequency ω is such that $R\omega \ll c$.

a) [10 pts] If $n = 0$, show that there is magnetic dipole radiation but no electric dipole radiation.

b) [10 pts] If $n = 1$, show that there is electric dipole radiation but no magnetic dipole radiation.

c) [10 pts] If $n = 2$, show that there is neither electric dipole nor magnetic dipole radiation. What happens in this case? What is the frequency of the emitted radiation? You must explain your answer, not just give a guess.

Some formulas that *might* be helpful, if you need them

Legendre Polynomials:

$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{1}{2}(3x^2 - 1),\ P_3(x) = \frac{1}{2}(5x^3 - 3x)$

Electric Dipole Radiation:

For a charge distribution with oscillating dipole moment $\mathbf{p}(t) = \mathrm{Re}[\mathbf{p}_\omega \mathrm{e}^{-i\omega t}]$, the electric dipole approximation for the electric and magnetic fields in the radiation zone gives,

$$\begin{aligned}
\mathbf{E}(\mathbf{r}, t) &= \mathrm{Re}\left[-k^2 \frac{\mathrm{e}^{i(kr-\omega t)}}{r} \hat{r} \times (\hat{r} \times \mathbf{p}_\omega)\right] \\
\mathbf{B}(\mathbf{r}, t) &= \mathrm{Re}\left[k^2 \frac{\mathrm{e}^{i(kr-\omega t)}}{r} \hat{r} \times \mathbf{p}_\omega\right]
\end{aligned}$$

where $k = \omega/c$.