PHY 415 **Midterm Exam** **Fall 2017**

1) [40 points total]

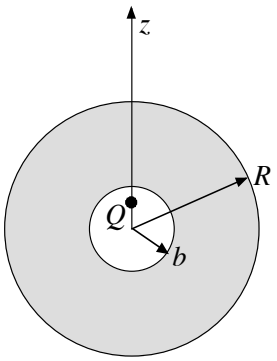
In each of the following parts, you should be able to give an answer *without* any detailed calculation. But you must make sure that your answer and the reason behind it is *clearly* explained, you cannot just guess!

a) [10 pts] An electrostatic potential is given in spherical coordinates by the expression below. Describe the charge distribution that produced this potential.

$$\varphi(r, \theta, \varphi) = \frac{a}{r} + \frac{b \cos\theta}{r^2}$$

b) [10 pts] Explain why Maxwell's correction to Ampere's Law is necessary for charge conservation.

c) [10 pts] A conducting sphere of radius R is connected to a battery which keeps it at a constant potential ϕ_0, relative to a reference point at infinity (i.e. $\phi \to 0$ as $r \to \infty$). Inside the sphere is a concentric hollow spherical cavity of radius b, and inside the cavity is a charge Q, positioned a distance s from the origin as shown in the sketch below.



Is there a net force on Q? If so, what is the physical origin of this force, and in what direction would it cause Q to move? What is the electric field $\mathbf{E}$ at a distance r along the $\hat{\mathbf{z}}$ axis, when $r \gg R$?

d) [10 pts] Consider a thin square of side a lying in the xy plane at $z = 0$, centered about the origin. The square has a uniform surface charge density σ_0. Consider the electric field $\mathbf{E}$ at a point at height z on the z-axis.

(i) Write an approximate expression for $\mathbf{E}$ when $z \gg a$.

(ii) Write an approximate expression for $\mathbf{E}$ when $z \ll a$.

2) [30 points total]

A thin spherical shell of radius R carries a surface charge density $\sigma(\theta) = \sigma_0 \sin^2\theta$, where θ is the usual polar angle in spherical coordinates.

a) [22 pts] Find the electrostatic potential $\phi(\mathbf{r})$ both inside the shell ($r < R$) and outside the shell ($r > R$).

b) [8 pts] Taking the origin at the center of the shell, does this configuration have a non-zero monopole moment? non-zero dipole moment? non-zero quadrupole moment? higher moments? You must clearly and convincingly explain your answer, not just say "yes" or "no."

---

3) [30 points]

A thin circular disk of radius R, lying in the xy plane at $z = 0$ and centered at the origin, has on it a fixed surface charge density

$$\sigma(r, \varphi) = Ar \sin(2\varphi)$$

where r and φ are the usual polar coordinates in the xy plane.

Compute the approximate electrostatic potential $\phi(\mathbf{r})$ of this disk up through the electric quadrupole term. Express your answer in spherical coordinates.