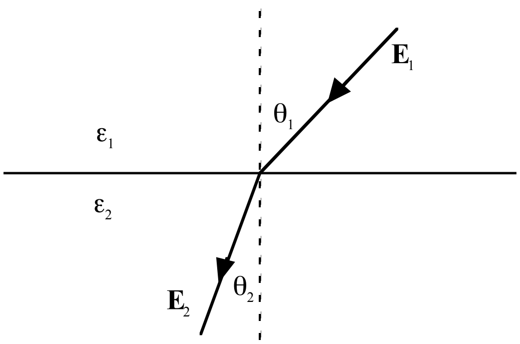
PHY 415 **Final Exam** **Fall 2018**

1) [25 points total]

a) [13 pts] A point charge $q > 0$ is positioned a large distance r from a neutral atom of atomic polarizability α. How does the force between the charge and the atom vary with distance r? Is the force repulsive or attractive? What happens if now $q < 0$?

b) [12 pts] Consider two semi-infinite dielectrics with real positive dielectric constants ε_1 and ε_2 that meet at a plane interface as in the figure below. If a *static* uniform electric field $\mathbf{E}_1$ is present in dielectric 1, making an angle θ_1 with respect to the normal to the interface, then what is the angle θ_2 of the static uniform electric field $\mathbf{E}_2$ in dielectric 2? Assume that there is no free charge at the interface. Write your answer for θ_2 in terms of θ_1, ε_1 and ε_2.



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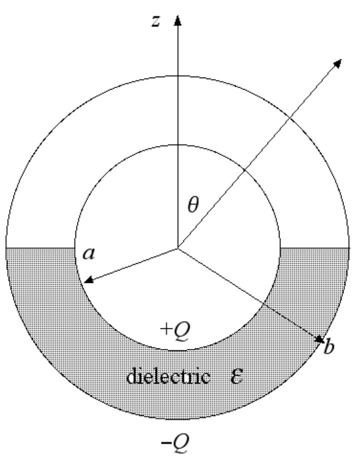
2) [25 points total]

Two thin concentric conducting spherical shells of inner radius a and outer radius b carry total free charges $+Q$ and $-Q$ respectively. The space between them is half filled by a dielectric material with dielectric constant ε, as sketched below.



a) [9 pts] [Find the electric field $\mathbf{E}$ everywhere between the shells, $a < r < b$.

b) [8 pts] Calculate the total surface charge density $\sigma_{\text{tot}}(\theta)$ on the surface of the inner shell at $r = a$.

c) [8 pts] Calculate the induced bound surface charge density $\sigma_b(\theta)$ on the surface of the dielectric at $r = a$.

3) [25 points total]

a) [9 pts] Consider the propagation of a linearly polarized plane transverse electromagnetic wave traveling in the $\hat{\mathbf{z}}$ direction in a dielectric material. In lecture we considered behavior as a function of the wave frequency ω and found that behavior could be considered to belong to one of three different regimes: transparent propagation, resonant absorption, and total reflection. Describe the characteristic feature of the complex dielectric function $\varepsilon(\omega)$, and the complex wavevector k, in each of these three regimes.

b) [6 pts] Consider the *effective* dielectric function $\varepsilon(\omega)$ of a conductor, and explain qualitatively what is the difference between this and the $\varepsilon(\omega)$ of a dielectric insulator. What is a clear physical example of a difference in behavior between the conductor and the dielectric, with respect to electromagnetic plane wave propagation.

c) [10 pts] Consider a linearly polarized plane transverse electromagnetic wave traveling in the $\hat{\mathbf{z}}$ direction with frequency ω in the region of total reflection of either a dielectric or conductor. Compute the time averaged Poynting vector*

$$\langle \mathbf{S}(\mathbf{r},t)\rangle = \frac{c}{4\pi}\,\langle \mathbf{E}(\mathbf{r},t)\times \mathbf{H}(\mathbf{r},t)\rangle$$

and explain why your result is consistent with this being a region of total reflection. For this calculation you may assume that μ is a real constant.

*In lecture we mentioned some difficulties with using this formula for $\mathbf{S}$ in a general situation, but it should be ok when talking about waves of a single frequency ω.

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4) [25 points total]

Consider the radiation emitted by a thin circular wire loop of radius R, centered about the origin in the xy plane at $z = 0$. The current flowing in the loop is given by

$$I(\varphi,t) = \mathrm{Re}\left[I_0\cos(n\varphi)\mathrm{e}^{-i\omega t}\right]$$

where φ is the usual azimuthal angle in spherical coordinates (i.e. the angle in the xy plane). The frequency ω is such that $R\omega \ll c$.

a) [8 pts] If $n = 0$, show that there is magnetic dipole radiation but no electric dipole radiation.

b) [8 pts] If $n = 1$, show that there is electric dipole radiation but no magnetic dipole radiation.

c) [9 pts] If $n = 2$, show that there is neither electric dipole nor magnetic dipole radiation. What happens in this case? You must do a calculation or give a convincing argument to support your answer, not just give a guess.

Hint: For all parts, think about what the charge in the loop is doing. If you argue convincingly, you don't need to do an involved calculation.