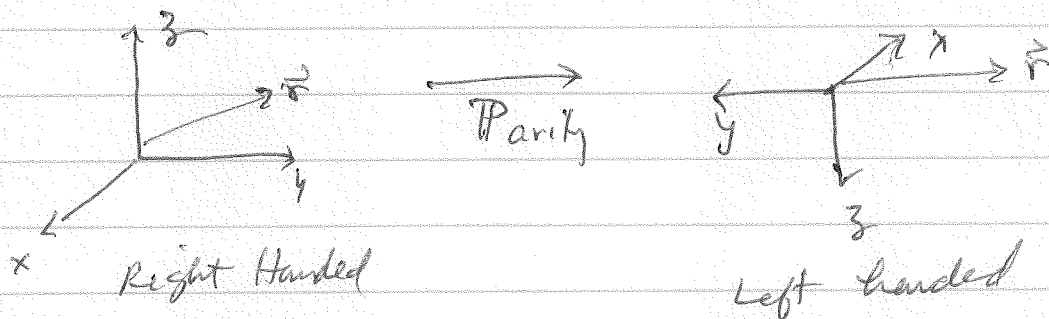


Symmetry under parity transformation vector vs. pseudo vector



$$\vec{r} = (x, y, z) \rightarrow (-x, -y, -z)$$

$$P(\vec{r}) = -\vec{r} \quad \text{position } \vec{r} \text{ is odd under parity}$$

Any vector-like quantity that is odd under P is a vector.

examples of vectors

position \vec{r}

velocity $\vec{v} = \frac{d\vec{r}}{dt}$

acceleration $\vec{a} = \frac{d\vec{v}}{dt}$

since \vec{r} is vector and t is scalar
 $P(t) = t$

Force $\vec{F} = m\vec{a}$

momentum $\vec{p} = m\vec{v}$

since \vec{a} is vector and m is scalar

since \vec{v} is vector and m is scalar

electric field $\vec{F} = q\vec{E}$

since \vec{F} is vector and q is scalar

$$P(q) = q$$

current $\vec{j} = \sum_i q_i \vec{v}_i \delta(\vec{r} - \vec{r}_i(t))$

any vector-like quantity that is even under \mathcal{P} is a pseudovector

angular momentum $\vec{L} = \vec{r} \times \vec{p}$ since $\vec{r} \rightarrow -\vec{r}$ and $\vec{p} \rightarrow \vec{p}$,
 $\vec{L} \rightarrow \vec{L}$ under \mathcal{P}

\vec{L} is even under \mathcal{P}

magnetic field $\vec{F} = g \vec{v} \times \vec{B}$

since \vec{F} and \vec{v} are vectors and g is scalar, \vec{B} must be pseudovector,
pseudovector.

cross product of any two vectors is a pseudovector

" " " vector and pseudovector is a vector

when solving for \vec{E} , it can only be made up of vectors that exist in the problem

when solving for \vec{B} , it can only be made up of pseudovectors that exist in the problem

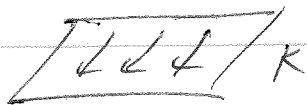
ex charged plane



only directions in problem is normal \hat{m}
 \hat{m} is a vector

$$\vec{E} \propto \hat{m}$$

surface current



only directions are the vectors \hat{m} and \vec{K} . But \vec{B} can only be made of pseudovectors

$$\Rightarrow \vec{B} \propto (\vec{K} \times \hat{m})$$

Dielectrics + Magnetic Materials - Macroscopic Maxwell Equ

Dielectrics

Maxwell's equations apply exactly to the true microscopic electric and magnetic fields that arise from all charges and currents.

$$\vec{\nabla} \cdot \vec{b} = 0 \quad \vec{\nabla} \times \vec{e} + \frac{1}{c} \frac{\partial \vec{b}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{e} = 4\pi \rho_0 \quad \vec{\nabla} \times \vec{b} = \frac{4\pi}{c} \vec{j}_0 + \frac{1}{c} \frac{\partial \vec{e}}{\partial t}$$

where \vec{e} and \vec{b} are microscopic fields from total charge density ρ_0 and current density \vec{j}_0 .

However, in most problems involving macroscopic objects, if we took ρ_0 and \vec{j}_0 to describe charge + current of each individual atom in a material, then they, and the resulting \vec{e} and \vec{b} would be enormously complicated functions varying rapidly over distances $\sim 10^{-8}$ cm and times $\sim 10^{-16}$ sec.

In classical EM we are generally concerned with phenomena that vary extremely slowly compared to these length + time scales,

Rather than worry about the microscopic details of ρ and \vec{j} and resulting \vec{E} and \vec{B} we want to describe phenomena in terms of averaged ~~smoothly varying~~ ~~quantities~~ averaged quantities that are smoothly varying at the atomic scale.

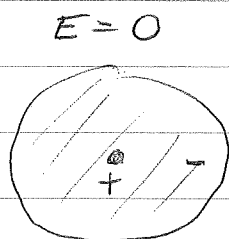
This results in what are known as the macroscopic Maxwell equations.

Dielectric Materials

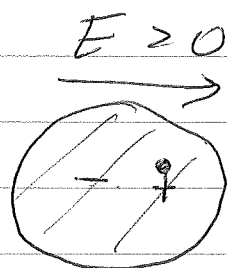
can be solid, liquid or gas

A dielectric material is an insulator. Electrons are bound to the ionic cores of the atoms.

When no electric field is present, the averaged ρ in the dielectric vanishes! One might therefore think that electrostatics in a dielectric is just due to whatever "extra" or "free" charge is added to the dielectric. However this is not true due to the phenomena of "polarization".



electron cloud centered on ionic ~~core~~ core
dipole moment vanishes



electron cloud and ionic core displaced $\vec{d} \propto \vec{E}$
atom is "polarized"
has dipole moment $\vec{p} = q\vec{d} \propto q\vec{E}$

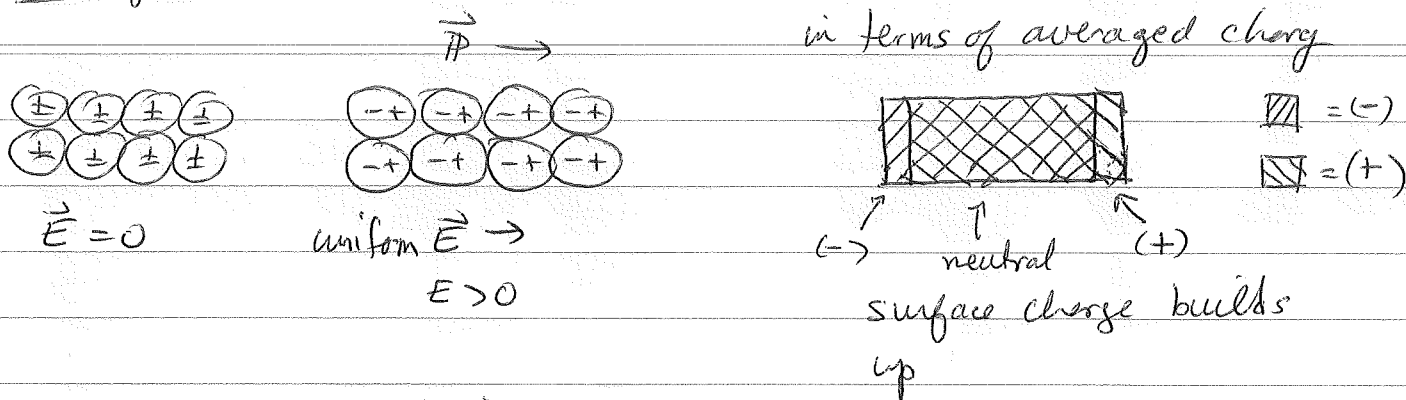
$\vec{p} = \alpha \vec{E}$
↑
atomic polarizability

Polarization density $\vec{P}(\vec{r}) = \sum_i \vec{p}_i \delta(\vec{r} - \vec{r}_i)$

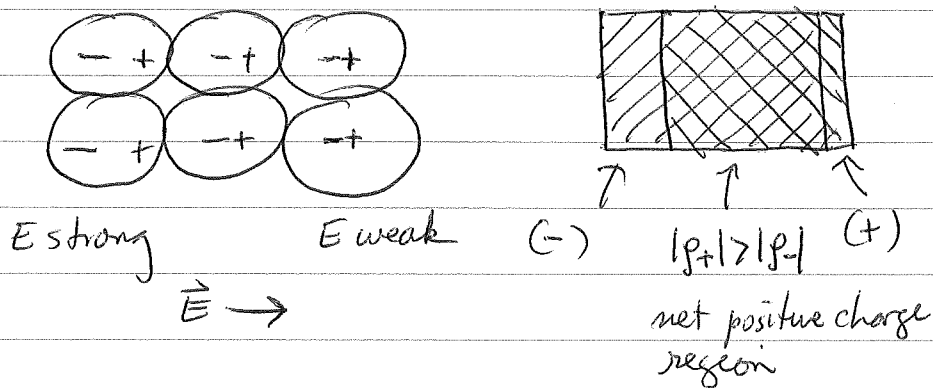
\vec{p}_i
dipole moment of atom i
at position \vec{r}_i

Polarization density \vec{P} can give rise to regions of net charge - sometimes called "bound charge"

Example



For a non uniform \vec{E} , atoms are more strongly polarized where E is largest



For uniform \vec{P} , build up surface charge σ_b
 For nonuniform \vec{P} , also can build up vol charge density ρ_b .

We now carry out the averaging explicitly to see how such polarization enters the macroscopic Maxwell equations

(Jackson 6.6)

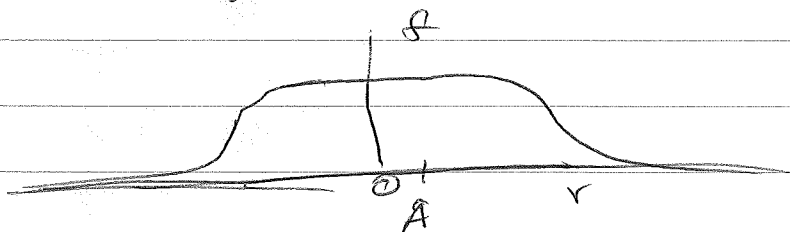
Define spatially averaged quantities by

$$\langle F(\vec{r}, t) \rangle = \int d^3r' f(\vec{r}') F(\vec{r} - \vec{r}', t)$$

where $f(\vec{r})$ vanishes for $|\vec{r}|$ large on microscopic length scales, but short on macroscopic length scales.

$f(\vec{r})$ normalized to unity $\int d^3r f(\vec{r}) = 1$.

Other details of $f(\vec{r})$ are not too important, as long as $f(\vec{r})$ is a smooth function of \vec{r}



want $f \approx 1$ for $r < R$

$f \approx 0$ for $r \gg R$

where R is length scale
in between micro + macro

$$\frac{\partial}{\partial r_i} \langle F(\vec{r}, t) \rangle = \int d^3r' f(\vec{r}') \frac{\partial F(\vec{r} - \vec{r}')}{\partial r_i} = \left\langle \frac{\partial F}{\partial r_i} \right\rangle$$

$$\frac{\partial}{\partial t} \langle F(\vec{r}, t) \rangle = \left\langle \frac{\partial F}{\partial t} \right\rangle$$

Define the macroscopic fields

$$\vec{E}(\vec{r}, t) \equiv \langle \vec{e}(\vec{r}, t) \rangle$$

$$\vec{B}(\vec{r}, t) \equiv \langle \vec{b}(\vec{r}, t) \rangle$$

$$\begin{aligned} \text{Then } \vec{\nabla} \cdot \vec{b} = 0 &\Rightarrow \langle \vec{\nabla} \cdot \vec{b} \rangle = 0 \\ &\Rightarrow \vec{\nabla} \cdot \langle \vec{b} \rangle = 0 \\ &\Rightarrow \vec{\nabla} \cdot \vec{B} = 0 \end{aligned}$$

$$\vec{\nabla} \times \vec{e} + \frac{\partial \vec{b}}{\partial t} = 0 \Rightarrow \vec{\nabla} \times \langle \vec{e} \rangle + \frac{\partial \langle \vec{b} \rangle}{\partial t} = 0$$

$$\Rightarrow \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

Remaining Maxwell eqn, upon averaging, become

$$\vec{\nabla} \cdot \vec{E} = 4\pi \langle \rho_0 \rangle$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \langle \vec{j}_0 \rangle + \frac{\partial \vec{E}}{\partial t}$$

Consider $\langle \rho_0 \rangle$

$$\rho_0 = \sum_i q_i \delta(\vec{r} - \vec{r}_i(t)) \quad \text{sum over all charges}$$

Consider dividing the charge into "free" charges and "bound" charges, where the latter are associated with the molecules that make up the dielectric

$$\rho_{\text{free}} = \sum_{i \text{ free}} q_i \delta(\vec{r} - \vec{r}_i(t)) \quad \begin{array}{l} \text{sum over only free charges} \\ \text{ie, external charge added to} \\ \text{the material} \end{array}$$

$$\rho_{\text{bound}} = \sum_n \rho_n(\vec{r}, t)$$

↑ charge distribution of molecule n

$$\rho_n(\vec{r}, t) = \sum_{i \in n} q_i \delta(\vec{r} - \vec{r}_i(t)) \quad \text{sum over charges in molecule n}$$

Define $q_n = \sum_{i \in n} q_i$ total charge molecule n

$\vec{p}_n = \sum_{i \in n} q_i \vec{r}_{ni}$ dipole moment about center of mass of molec n

$\vec{Q}'_n = \sum_{i \in n} 3 q_i \vec{r}_{ni} \vec{r}_{ni}$ quadrupole moment about center of mass of molec n

(prime on \vec{Q}' since definition here is a little different from that of multipole exp)
 \vec{Q}' is not traceless

$$\langle \rho_n(\vec{r}, t) \rangle = q_n f(\vec{r} - \vec{r}_n) - \vec{p}_n \cdot \vec{\nabla} f(\vec{r} - \vec{r}_n)$$

$$+ \frac{1}{6} \sum_{\alpha\beta} (Q'_n)_{\alpha\beta} \frac{\partial^2 f(\vec{r} - \vec{r}_n)}{\partial r_\alpha \partial r_\beta}$$

Now use $\langle \delta(\vec{r} - \vec{r}_n) \rangle = f(\vec{r} - \vec{r}_n)$ by definition of averaging

$$\Rightarrow \langle \rho_n(\vec{r}, t) \rangle = \langle q_n \delta(\vec{r} - \vec{r}_n) \rangle$$

$$- \vec{\nabla} \cdot \langle \vec{p}_n \delta(\vec{r} - \vec{r}_n) \rangle \quad \vec{\nabla} \cdot p \delta = \vec{p} \cdot \vec{\nabla} \delta$$

$$+ \frac{1}{6} \sum_{\alpha\beta} \frac{\partial^2}{\partial r_\alpha \partial r_\beta} \langle (Q'_n)_{\alpha\beta} \delta(\vec{r} - \vec{r}_n) \rangle$$

Now

$$\langle \rho_{\text{bound}}(\vec{r}, t) \rangle = \sum_n \langle \rho_n(\vec{r}, t) \rangle$$

$$= \left\langle \sum_n q_n \delta(\vec{r} - \vec{r}_n) \right\rangle - \vec{\nabla} \cdot \left\langle \sum_n \vec{p}_n \delta(\vec{r} - \vec{r}_n) \right\rangle$$

$$+ \frac{1}{6} \sum_{\alpha\beta} \frac{\partial^2}{\partial r_\alpha \partial r_\beta} \left\langle \sum_n (Q'_n)_{\alpha\beta} \delta(\vec{r} - \vec{r}_n) \right\rangle$$

Define $\vec{P}(\vec{r}, t) \equiv \left\langle \sum_n \vec{p}_n \delta(\vec{r} - \vec{r}_n) \right\rangle$ average polarization density

$\vec{Q}'(\vec{r}, t) \equiv \frac{1}{6} \left\langle \sum_n \vec{Q}'_n \delta(\vec{r} - \vec{r}_n) \right\rangle$ average quadrupole density

$$\langle \rho_{\text{bound}} \rangle = \sum_n \langle q_n \delta(\vec{r} - \vec{r}_n) \rangle - \vec{\nabla} \cdot \vec{P} + \sum_{\alpha\beta} \frac{\partial^2}{\partial r_\alpha \partial r_\beta} \vec{Q}'_{\alpha\beta}$$

Define the macroscopic charge density

$$\rho \equiv \left\langle \sum_{i \text{ free}} q_i \delta(\vec{r} - \vec{r}_i) \right\rangle + \left\langle \sum_n q_n \delta(\vec{r} - \vec{r}_n) \right\rangle$$

then

↑ vanishes if molecule is charge neutral

$$\vec{\nabla} \cdot \vec{E} = 4\pi \langle \rho_0 \rangle = 4\pi \left[\rho - \vec{\nabla} \cdot \vec{P} + \sum_{\alpha\beta} \frac{\partial^2}{\partial r_\alpha \partial r_\beta} \vec{Q}'_{\alpha\beta} \right]$$

$$\sum_\alpha \frac{\partial}{\partial r_\alpha} \left[E_\alpha + 4\pi P_\alpha - 4\pi \sum_\beta \frac{\partial}{\partial r_\beta} \vec{Q}'_{\alpha\beta} \right] = 4\pi \rho$$

Define electric displacement vector

$$\vec{D}_\alpha = E_\alpha + 4\pi P_\alpha - 4\pi \sum_\beta \frac{\partial}{\partial r_\beta} \vec{Q}'_{\alpha\beta}$$

then $\boxed{\vec{\nabla} \cdot \vec{D} = 4\pi \rho}$

In most materials, the quadrupole and higher terms are negligible and we can take $\vec{D} = \vec{E} + 4\pi \vec{P}$