PHY 415 **Midterm Exam** **Fall 2019**

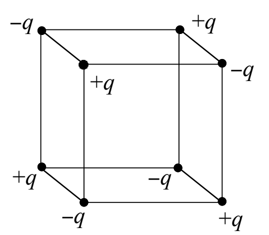
1) [60 points total]
Note: parts (a) - (c) are 10 points each, but (d) and (e) are 15 points each.

In each of the following situations, the electric field **E** or magnetic field **B** will decay, to leading order, as $1/r^n$ where r is the distance from the source to the observor. For each situation, give the value of n. You do not need to give a full calculation, but you must convincingly state the reason for the value of n in each case.

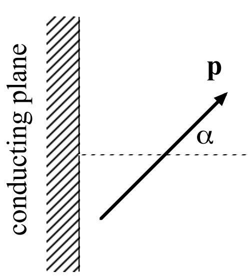
a) [10 pts] A thin spherical shell of radius R, is centered on the origin, with a surface charge density of $\sigma(\theta) = \sigma_0 \cos^3(\theta)$. Here σ_0 is a constant, and θ is the usual spherical polar angle.

b) [10 pts] A thin circular disk of radius R in the xy plane at $z = 0$, is centered on the origin, with surface charge density $\sigma(r) = C \sin(2\pi r/R)$. Here r is the radial distance from the center of the disk.

c) [10 pts] Eight point charges are located at the corners of a cube with side of length a, as shown below. The charges are all of equal magnitude, but alternate in sign as one goes around any of the faces of the cube.



d) [15 pts] An electric dipole **p** is oriented at an angle of α with respect to an infinite flat grounded conducting plane, as shown below. We are interested in the electric field far from the dipole, in front of the conducting plane. What is the value of n when $\alpha = 0°$, $45°$, $90°$?



e) [15 pts] Two circular wire loops, each of radius R, are centered about the origin in the xy plane, one at height $z = +d/2$, the other at height $z = -d/2$. The first has current I circulating counterclockwise, the second has current I circulating clockwise. Assume d and R are of comparable length. In addition to finding the value of n, explain how you would go about computing $\mathbf{B}(\mathbf{r})$ to get the leading $1/r^n$ behavior; you don't have to do this calculation, but you should clearly explain how you would do it.

2) [40 points total]

Consider a thin spherical shell of radius R, whose surface is fixed to the potential $\phi(R, \theta, \varphi) = \phi_0 \cos^3 \theta$, where θ and φ are the usual spherical angles.

a) [14 pts] What is the electrostatic potential $\phi^{in}(\mathbf{r})$ inside the sphere?

b) [14 pts] What is the electrostatic potential $\phi^{out}(\mathbf{r})$ outside the sphere?

c) [13 pts] What is the surface charge density $\sigma(\theta, \varphi)$ on the surface of the sphere?

Legendre Polynomials:

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{1}{2}(3x^2 - 1)$

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$