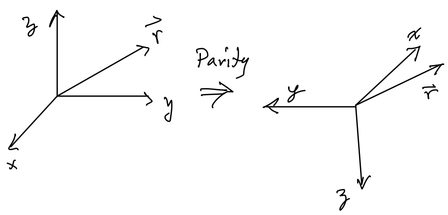
Unit 2-6: Symmetry Under Parity Transformation: vector vs pseudovector



Right Handed Left Handed coordinate systems

A Parity transformation inverts all the Cartesian axes, and takes one from a right handed to a left handed coordinate system. Under a parity transformation, the coordinates of the position vector change sign,

$$\mathbf{r} = (x, y, z) \qquad \Rightarrow \qquad (-x, -y. - z) \tag{2.6.1}$$

If $\mathbb{P}$ is the parity operator, we can write $\mathbb{P}(\mathbf{r}) = -\mathbf{r}$, and we say that $\mathbf{r}$ is *odd* under parity (i.e. it changes sign).

Any vector-like quantity that is odd under a parity transformation we will call a vector.

examples of vectors:

position $\mathbf{r}$

velocity $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$, since $\mathbf{r}$ is vector and t is a scalar under parity, $\mathbb{P}(t) = t$

acceleration $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$, since $\mathbf{v}$ is a vector and t is a scalar

Force $\mathbf{F} = m\mathbf{a}$, since $\mathbf{a}$ is a vector and m is a scalar

momentum $\mathbf{p} = m\mathbf{v}$, since $\mathbf{v}$ is a vector and m is a scalar

electric field $\mathbf{E} = \mathbf{F}/q$, since $\mathbf{F}$ is a vector and q is a scalar

current $\mathbf{j} = \sum_i q_i \mathbf{v}_i \delta(\mathbf{r} - \mathbf{r}_i)$, since $\mathbf{v}_i$ is a vector and q_i and $\delta(\mathbf{r})$ are scalars

In contrast, any vector-like quantity that is *even* (i.e. does *not* change sign) under parity is called a pseudovector.

angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, since under parity $\mathbf{r} \to -\mathbf{r}$ and $\mathbf{p} \to -\mathbf{p}$, then $\mathbf{L} \to \mathbf{L}$ under parity. $\mathbf{L}$ is even under parity, so $\mathbf{L}$ is a pseudovector!

magnetic field $\mathbf{F} = q\dfrac{\mathbf{v}}{c} \times \mathbf{B}$, since $\mathbf{F}$ and $\mathbf{v}$ are vectors, and q is a scalar, $\mathbf{B}$ must be a pseudovector.

A cross product of any two vectors is a pseudovector. A cross product of any vector with a pseudovector is a vector.

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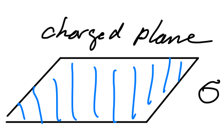
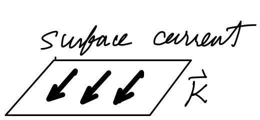


For a charged plane, the only direction in the problem is the normal $\hat{\mathbf{n}}$, which is a vector $\Rightarrow$ $\mathbf{E} \propto \hat{\mathbf{n}}$



For a planar surface current, the only directions in the problem are the vectors $\hat{\mathbf{n}}$ and $\mathbf{K}$. But $\mathbf{B}$ is a pseudovector and can only be made up of pseudovectors $\Rightarrow$ $\mathbf{B} \propto (\mathbf{K} \times \hat{\mathbf{n}})$