Phy 418 **Final Exam** **Spring 2005**

1) [35 points]

Consider atoms A that can bind together to form a diatomic molecule A_2,

$$2A \leftrightarrow A_2 \ .$$

The binding energy of the molecule is Δ. Assume that the atoms and the diatomic molecules can be treated as ideal, indistinguishable, classical point particles (i.e. ignore any rotational, vibrational, or electronic excitations). Suppose that there are initially N atoms A and no molecules A_2 confined to a cubic box of volume V. What will be the ratio of the number of atoms A to the number of molecules A_2 when the system is in equilibrium at a temperature T?

2) [33 points]

Consider a one-dimensional quantum harmonic oscillator, of angular frequency ω, at thermal equilibrium at temperature T.

a) Find an expression for the entropy of the oscillator.

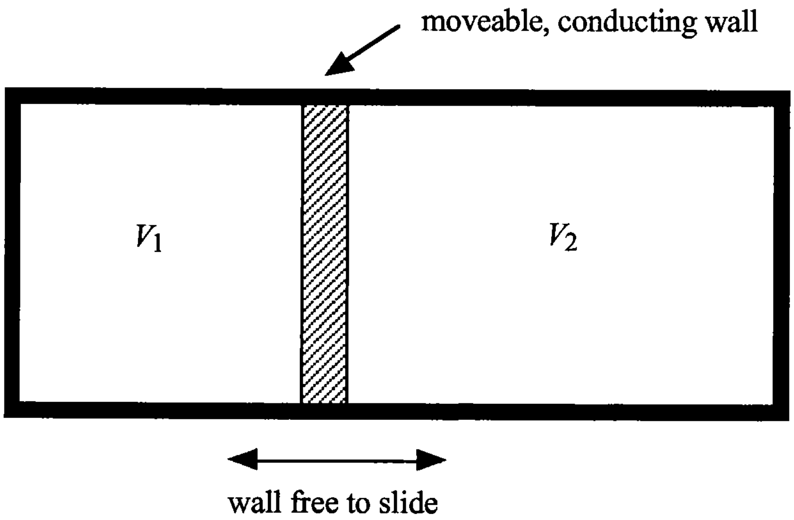
b) Find expressions which give the approximate temperature dependence of the entropy for both low ($k_B T << \hbar\omega$) and high ($k_B T >> \hbar\omega$) temperatures.

c) Consider a system of N such oscillators with frequencies ω distributed with equal probability over the range $0 \leq \omega \leq \omega_0$. Find the specific heat of the system for both low and high temperatures.

[over]

3) [32 points]

Consider a thermodynamic system consisting of two gases in volumes V_1 and V_2, separated by a thermally conducting, freely moving wall, as shown in the diagram below. The gas in V_1 has N_1 particles, the gas in V_2 has N_2 particles, and particles *cannot* pass through the separating wall. The system is isolated from the rest of the universe, and the total volume $V_1 + V_2 = V$ is fixed. The system is in thermal equilibrium at temperature T.



a) Give a derivation showing that in equilibrium the pressures of the two gases must be equal.

If the system is initially in equilibrium, for each case below, explain in which direction the wall between the two gases will move, if the temperature is increased a small amount ΔT.

b) The gas in V_1 is an ideal gas of fermions in the degenerate limit, i.e. $k_B T \ll \varepsilon_F$. The gas in V_2 is a classical ideal gas.

c) The gas in V_1 is an ideal gas of bosons in a Bose-Einstein condensed state, i.e. $T < T_c$. The gas in V_2 is a classical ideal gas.

d) The gas in V_1 is an ideal gas of fermions, and the gas in V_2 is an ideal gas of bosons. However both are now in the non-degenerate limit where their behavior is approaching that of a classical ideal gas (i.e. keep the leading quantum correction to the classical equation of state).