1) [50 points]

Consider a classical gas of N non-interacting, indistinguishable, diatomic molecules confined to a volume V in equilibrium at temperature T. Each molecule is composed of two atoms and has a total mass m. Each atom has a quantized spin which can take on only one of two values, $S = +1/2$ or $S = -1/2$. Assume that you can ignore all other internal degrees of freedom of the molecule except for the two spins (i.e. you can ignore rotational, vibrational, and electronic degrees of freedom). The single particle Hamiltonian H of such a molecule in an applied magnetic field h is,

$$H = \frac{p^2}{2m} - JS_1S_2 - h(S_1 + S_2)$$

where $\mathbf{p}$ is the momentum of the molecule, S_1 and S_2 are the spins of the two atoms, h is the magnetic field, and J is a constant.

a) Calculate the Helmholtz free energy of the gas, $A(T, V, N, h)$.

b) Calculate the average magnetization M of the gas,

$$M = \left\langle \sum_{i=1}^{N} (S_{i1} + S_{i2}) \right\rangle$$

where S_{i1} and S_{i2} are the spins of molecule i.

c) Calculate the specific heat at constant volume, C_V, and the specific heat at constant pressure, C_p.

2) [50 points]

Consider a classical gas of N non-interacting, indistinguishable, particles in the grand canonical ensemble with fugacity $z = e^{\beta\mu}$.

a) Show that the probablity $p(N)$ that there are exactly N particles in the system follows a Poisson distribution:

$$p(N) = \frac{\lambda^N}{N!} e^{-\lambda}$$

where $\lambda = \langle N \rangle$ is the average number of particles.

b) Find expressions for $\langle N \rangle$ and $\langle N^2 \rangle$ in terms of the fugacity z and the single particle partition function.