PHY 418 **Final Exam** **Spring 2006**

1) [30 points total] Consider a classical gas of N identical, non-relativistic, non-interacting atoms confined to a box of volume V in equilibrium at a temperature T. Each atom can be considered as a point particle, however it has an internal degree of freedom that can be in one of two possible states with energies $\varepsilon_0 = 0$ and $\varepsilon_1 > 0$ respectively.

a) [10 pts] Prove that the equation of state for the gas remains the familiar ideal gas law.

b) [10 pts] Find the chemical potential of the gas.

c) [10 pts] Find the specific heat per particle at constant volume of the gas and sketch it as a function of temperature.

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2) [35 points total] Consider a non-interacting gas of N identical non-relativistic spin ½ fermions, confined to move in a one-dimensional system of length L.

a) [12 pts] The density of states, $g(\varepsilon)$, is defined as the number of single particle states with energy ε per unit energy per unit length. Compute $g(\varepsilon)$ for the gas.

b) [11 pts] Compute the Fermi energy of the gas as a function of its density $n = N/L$.

c) [12 pts] Compute the pressure of the gas at $T = 0$ as a function of its density n.

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3) [35 points total] Consider a non-interacting gas of N identical spin zero bosons. Suppose that the energy-momentum relationship for these particles is given by $\varepsilon(\mathbf{p}) = A|\mathbf{p}|^s$, for some fixed positive numbers A and s. The dimensionality of the gas is the number d, i.e. the "volume" of the gas is $V = L^d$, for a system of length L. In the following parts, we are considering behavior in the thermodynamic limit of $V \to \infty$.

a) [15 pts] For what values of s and d will there exist Bose-Einstein condensation at sufficiently low temperature?

b) [5 pts] For the case that there is Bose-Einstein condensation, write an expression that gives how the condensate density n_o depends on the temperature T, the condensation temperature T_c, and the total density $n = N/V$.

c) [10 pts] Show that the pressure p is related to the energy density E/V by

$$p = \frac{sE}{dV}$$

d) [5 pts] Can a gas of photons in $d = 3$ undergo Bose-Einstein condensation?