PHY 418 **Final Exam** **Spring 2007**

1) [35 points total]
Consider atoms A that can bind together to form a diatomic molecule A_2,

$$2A \leftrightarrow A_2 \ \ .$$

The binding energy of the molecule is Δ (that is, the ground state of the molecule is an energy Δ *lower* than the ground state of the two uncombined atoms). Assume that the atoms and the diatomic molecule can be treated as ideal, indistinguishable, non-relativistic, classical point particles (i.e. ignore any rotational, vibrational, or electronic excitations). Suppose that there are initially N_0 atoms A and *no* molecules A_2 confined to a cubic box of volume V. What will be the ratio of the number of atoms A to the number of molecules A_2 when the system comes to equilibrium at a temperature T?

2) [30 points total]
Consider a point particle of mass m attached to a harmonic spring with spring constant $\kappa = m\omega_0^2$. Consider that the particle moves only in one dimension, labeled by the coordinate x. The particle is in equilibrium at a temperature T.

a) [12 points] Assume that the particle behaves clasically. What is the root-mean-squared fluctuation $\Delta x \equiv \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ of the particle about its average position $\langle x\rangle$?

b) [12 points] Assume that the particle must be treated quantum mechanically. What now is Δx? (Hint: recall that for the quantum harmonic oscillator, the expected value of the kinetic energy equals the expected value of the potential energy - this is the quantum virial theorem.)

c) [6 points] Show that your answer in (b) reduces to your answer in (a) in the appropriate limit.

3) [35 points total]
Consider an ideal (i.e. non-interacting) gas of N extremely relativistic spin 1/2 fermions, whose energy-momentum relationship is well approximate by $\epsilon(\mathbf{p}) = c|\mathbf{p}|$. The gas is confined to a three dimensional box of volume V and is in equilibrium at temperature T. We consider the thermodynamic limit of $V \to \infty$ keeping the particle density $n = N/V$ constant.

a) [12 points] The density of states $g(\epsilon)$ is defined as the number of single particle states with energy ϵ per unit energy per unit volume. Compute $g(\epsilon)$ for this gas.

b) [11 points] Compute the Fermi energy of the gas.

c) [12 points] Compute the pressure of the gas at $T = 0$.