PHY 418 **Midterm Exam** **Spring 2011**

1) [50 points total]

Consider a cylindrical column of cross-sectional area $\mathcal{A}$ and infinite height. The column is filled with classical ideal gas of N non-interacting, non-relativistic, indistinguishable point particles of mass m. The gas is in equilibrium at a fixed temperature T. The force of gravity acts upon the particles of gas in the column via the gravitational potential energy $U(z) = mgz$, where z is the height within the column, and the constant g is the gravitational acceleration at the surface of the Earth.

a) What is the density of the gas $n(z)$ at height z within the column? [20 pts]

b) What is the pressure of the gas $p(z)$ at height z within the column? [20 pts]

c) What is the chemical potential of the gas $\mu(z)$ at height z within the column? [10 pts]

2) [50 points total]

Consider an ideal gas of N non-interacting, non-relativistic, indistinguishable particles of mass m. Each particle has an internal structure consisting of two intrinsic magnetic moments. The magnetic moments interact with each other and with an external applied magnetic field $\vec{h} = h\hat{z}$, so that the one-body Hamiltonian of the particle is,

$$\mathcal{H}^{(1)} = \frac{|\vec{p}|^2}{2m} - Js_1s_2 - (s_1 + s_2)h$$

where $s_1 = \pm 1$ and $s_2 = \pm 1$ give the two possible directions of alignment the two magnetic moments with respect to the applied field $\vec{h}$, $J > 0$ is the positive coupling constant of the interaction between the moments, and $\vec{p}$ is the momentum of the particle. The gas is in equilibrium at a fixed temperature T, in a box of volume V. (Note, the two moments s_1 and s_2 within each particle are distinguishable degrees of freedom.)

a) What is the total average energy $\langle E \rangle$ of the gas? [20 pts]

b) What is the average magnetic moment $\langle M \rangle \equiv \langle s_1 + s_2 \rangle$ of a particle of the gas? [20 pts]

c) What is the magnetic susceptibility of a particle of the gas, $\chi \equiv \lim_{h\to 0} \left(\partial \langle M \rangle / \partial h\right)_{T,V,N}$? [10 pts]