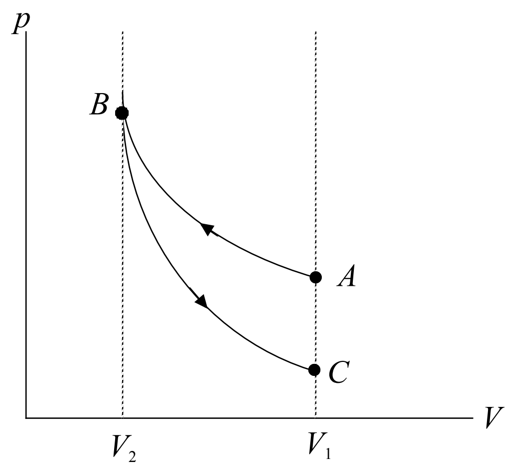
PHY 418 **Midterm Exam** **Spring 2012**

1) [20 points] Each quantity in the first row is equal to one quantity in the second row, via a Maxwell relation. Match the quantities in the first row to those in the second row, and write the correct second derivative of the appropriate thermodynamic potential that gives the Maxwell relation involved. When you specify the thermodynamic potential involved, be sure to write the variables that the potential depends on.

$$(a)\ -\left(\frac{\partial p}{\partial N}\right)_{T,V} \quad (b)\ -\left(\frac{\partial S}{\partial p}\right)_{T,N} \quad (c)\ \left(\frac{\partial V}{\partial N}\right)_{S,p}$$

$$(d)\ \left(\frac{\partial \mu}{\partial p}\right)_{S,N} \quad (e)\ \left(\frac{\partial \mu}{\partial V}\right)_{T,N} \quad (f)\ \left(\frac{\partial V}{\partial T}\right)_{p,N}$$

2) [20 points] Consider a classical ideal gas of N point particles, that is compressed and then expanded, as shown in the diagram below. The gas starts at point A in the $p-V$ diagram, with volume V_1 at temperature T_1. It is then *isothermally* compressed (i.e. at constant temperature) to volume V_2, denoted point B on the diagram. The gas is then *adiabatically* expanded (i.e. at constant entropy) back to volume V_1, denoted point C on the diagram. What is the final temperature T_2 at point C? Express your answer in terms of T_1, V_1, and V_2.

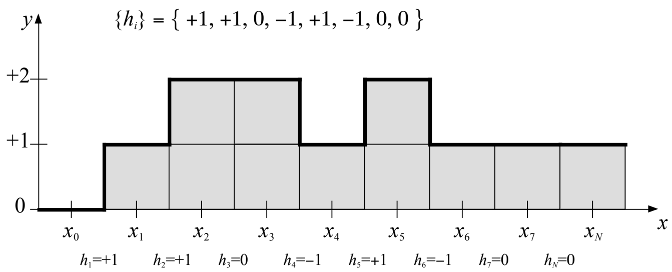


It might help you to know that the entropy S of an ideal gas is

$$S(E,V,N) = \left(\frac{N}{N_0}\right) S_0 + N k_B \ln\left[\left(\frac{E}{E_0}\right)^{3/2}\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-5/2}\right]$$

where E, V, N are the total energy, total volume, and total number of particles, and E_0, V_0, N_0, S_0 are constants.

4) [60 points total] Consider the following simple model for the surface of a two dimensional crystal of length $(N+1)a$, with a the fixed lattice constant, and N even. We will measure length in units such that $a = 1$. At each horizontal position x_i, $i = 0, \dots, N$, the height of the surface is denoted y_i. Going from x_{i-1} to x_i the surface can change its height only in steps with height difference $[y_i - y_{i-1}] \equiv h_i = 0, \pm 1$. We pin the height at x_0 to be $y_0 = 0$, so the height of the surface above position x_i is just $y_i = \sum_{j=1}^{i} h_j$. An example of a particular surface configuration is shown below.



If the energy per unit length of the surface is ϵ, then the total energy of a surface configuration specified by a particular set of values $\{h_i\}$ is,

$$E(\{h_i\}) = \epsilon \sum_{i=1}^{N} |h_i| + E_0 \qquad \text{where} \quad E_0 = \epsilon(N+1)$$

Consider the surface in equilibrium at a fixed temperature T.

a) [10 pts] What is the average energy $\langle E \rangle$ of the surface? Check that your answer makes sense in the limits $T \to 0$ and $T \to \infty$.

b) [10 pts] What is the root mean square average height $\sqrt{\langle y_N^2 \rangle}$ of the surface above position x_N?

c) [10 pts] What is the entropy of the surface $S(T)$?

d) [15 pts] Suppose that we now require the surface to be on average flat, i.e. we constrain the $\{h_i\}$ so that $y_N = \sum_{i=1}^{N} h_i = 0$. What is the entropy $S(E)$ for a surface with fixed total energy E?

e) [10 pts] What is the temperature $T(E)$ of the surface described in part (d)?

f) [5 pts] How does the entropy in part (d) compare with that in part (c)?

Hint: The multinomial distribution $N!/[M_1!M_2!M_3! \dots M_n!]$, with $\sum_{i=1}^{n} M_i = N$, tells you how many ways there are to divide N objects among n boxes, so that box i has M_i objects.