PHY 418 **Final Exam** **Spring 2013**

Answer all questions. Please put a box around your final answer for each part.

1) [35 points] Consider atoms A that can bind together to form a diatomic molecule A_2

$$A + A \leftrightarrow A_2$$

The binding energy of the molecule is Δ (i.e. the ground state energy of the molecules is Δ *lower* than that of the two atoms in their ground states). Assume that the atoms and the diatomic molecules can be treated as ideal, indistinguishable, *classical* point particles. Suppose that there are initially N atoms A and no molecules A_2 confined to a cubic box of volume V. What will be the ratio of the number of atoms A to the number of molecules A_2 when the system comes to equilibrium at temperature T?

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2) [35 points] Consider an ideal gas of N non-interacting, extremely relativistic, spin 1/2 fermions, whose energy-momentum relationship is well approximated by $\epsilon(\mathbf{p}) = c|\mathbf{p}|$. The particles are confined to a cubic three dimensional box of fixed volume $V = L^3$. The density of the gas is $n = N/V$, and we consider the thermodynamic limit of $V \to \infty$ with n held constant.

a) Find the density of states per unit energy per unit volume, $g(\epsilon)$, for the gas.

b) Find the Fermi energy of the gas.

c) Compute the pressure of the gas at $T = 0$.

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3) [30 points] In Problem 3 of Problem Set 7 you showed that, in the grand canonical ensemble, the fluctuation in the total number of particles in an ideal (i.e. non-interacting) quantum gas was,

$$\langle N^2 \rangle - \langle N \rangle^2 = \sum_i \left[\langle n_i^2 \rangle - \langle n_i \rangle^2 \right] = \sum_i \langle n_i \rangle (1 \pm \langle n_i \rangle)$$

where n_i is the occupation number of particles in single particle eigenstate i with energy ϵ_i. In the righthand term above, the $+$ sign is for a gas of bosons, while the $-$ sign is for a gas of fermions.

For free particles in a three dimensional cubic box of side length L, in equilibrium at temperature T, compute the relative fluctuation in the number of particles, $\frac{\sqrt{\langle N^2 \rangle - \langle N \rangle^2}}{\langle N \rangle}$, and show how it behaves to leading order as $\langle N \rangle \to \infty$, as a function of T,

a) for a gas of fermions in the degenerate limit. How does this compare to what you would expect for a classical gas of particles?

b) for a gas of bosons at a temperature below the Bose-Einstein condensation temperature. What is strange about your result?