

Please cross out any work you do not wish me to look at, and place a box around your final answer for each part.

1) [50 points total] You must explain your answer completely for all parts!

A box is partitioned by a wall into two parts, the right side and the left side. Each side is filled with an identical classical ideal gas of non-interacting, non-relativistic, indistinguishable point particles of mass  $m$ . The gas on each side of the box has the same number of particles  $N$ . The volume of the left side of the box is  $V_1$  and the volume of the right side of the box is  $V_2$ . The wall separating the two sides of the box is adiabatic (does not conduct heat), immovable, and non-porous. Initially the gas on the left side is in equilibrium at temperature  $T_1$ , while the gas on the right side is in equilibrium at temperature  $T_2$ . The wall is now removed and each gas is free to fill the entire volume. The system then comes into its new state of equilibrium.

a) [5 pts] Is the final entropy of the total system larger or smaller than the initial total entropy?

b) [10 pts] What is the final temperature  $T_f$  of the gas?

c) [15 pts] What is the final total pressure  $p_f$  of the gas? Express  $p_f$  only in terms of  $T_1$  and  $T_2$  and the initial pressures of the two sides  $p_1$  and  $p_2$ . If initially we had  $T_1 = T_2$ , is  $p_f$  greater or smaller than the average initial pressure  $\bar{p} = (p_1 + p_2)/2$ ?

d) [20 pts] Compute the change in total entropy  $\Delta S$  that results from removing the wall. Express your answer in terms of the variables  $T_1$ ,  $T_2$ ,  $V_1$ ,  $V_2$  and  $N$  only. Show that  $\Delta S = 0$  if  $V_1 = V_2$  and  $T_1 = T_2$ .

2) [50 points total]

Consider a classical ideal gas of  $N$  non-interacting, non-relativistic, indistinguishable atoms of mass  $m$ , confined to a box of volume  $V$  and in equilibrium at temperature  $T$ . Each atom  $i$  has a net spin  $s_i$  which can be in one of three possible states,  $s_i = -1, 0, +1$ . The magnetic moment produced by this spin interacts with an external magnetic field  $\mathbf{h} = h\hat{z}$  giving a contribution to the atom's energy,  $\epsilon_i = -\mu h s_i$ .

a) [15 pts] Find the canonical partition function  $Q_N(T, V, h)$ .

b) [5 pts] Find the pressure  $p$  as a function of  $T, V, N$  and  $h$ .

c) [10 pts] Find the average total energy  $\langle E \rangle$  of the gas as a function of  $T, V, N$  and  $h$ .

d) [10 pts] Find the average total magnetization  $\langle M \rangle = \mu \sum_{i=1}^N \langle s_i \rangle$  of the gas as a function of  $T, V, N$  and  $h$ .

e) [5 pts] Find the chemical potential  $\mu$  of the gas as a function of  $T, V, N$  and  $h$ .