Ideal Bose Gas

Bose-Einstein Condensation

Bose occupation function

\[ n(c) = \frac{1}{z^{-1}e^{\beta c} - 1} \]

We had for the density of an ideal (non-interacting) Bose gas

\[ \frac{N}{V} = \frac{1}{V} \sum_{k} \frac{1}{z^{-1}e^{\beta E(k)} - 1} = \frac{1}{(2\pi)^3} \int_{0}^{\infty} dk \frac{4\pi k^2}{2\hbar^2} \frac{1}{z^{-1}e^{\beta \frac{\hbar^2 k^2}{2m}} - 1} \]

Recall, we need \( z \leq 1 \) for the occupation number at \( E(k=0) = 0 \) to remain positive \( M(0) \geq 0 \)

\[ M(0) = \frac{1}{z^{-1} - 1} = \frac{z}{1 - z} \Rightarrow z \leq 1 \quad z = e^{\beta \mu} \Rightarrow M \leq 0 \]

Substitute variables \( y = \frac{\beta \frac{\hbar^2 k^2}{2m}}{2\hbar^2} \Rightarrow k = \sqrt{\frac{2my}{\beta \hbar^2}} \)

\[ dk = \frac{2my}{\beta \hbar^2} \frac{dy}{2y} \]

\[ \Rightarrow \frac{N}{V} = \left( \frac{2m}{\beta \hbar^2} \right)^{3/2} \frac{4\pi}{(2\pi)^3} \frac{1}{2} \int_{0}^{\infty} dy \frac{y^{1/2}}{z^{-1}e^{y} - 1} \]

\[ \frac{N}{V} = \frac{1}{2^3} g_{3/2}(z) \quad \text{where} \quad \Delta = \left( \frac{\hbar^2}{2\pi m k_B T} \right)^{1/2} \text{thermal wavelength} \]

\[ g_{3/2}(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} dy \frac{y^{1/2}}{z^{-1}e^{y} - 1} \]
Consider the function

$$g_{3/2}(z) = \frac{2}{\sqrt{\pi}} \int_0^\infty dy \frac{y^{1/2}}{e^y - 1} = z + \frac{z^2}{2^{3/2}} + \frac{z^3}{3^{3/2}} + \cdots$$

$g_{3/2}(z)$ is a monotonic increasing function of $z$ for $z \leq 1$.

As $z \to 1$, $g_{3/2}(z)$ approaches a finite constant

$$g_{3/2}(1) = 1 + \frac{1}{2^{3/2}} + \frac{1}{3^{3/2}} + \cdots = \zeta(3/2) \approx 2.612$$

This is the Riemann zeta function.

We can see that $g_{3/2}(1)$ is finite as follows:

$$g_{3/2}(1) = \frac{2}{\sqrt{\pi}} \int_0^\infty dy \frac{y^{1/2}}{e^y - 1}$$

as $y \to \infty$ the integral converges. Integral is largest at small $y$.

(recall small $y$ corresponds to low energy where $m(y)$ is largest)

For small $y$, we can approximate

$$\int_0^y \frac{dy}{e^{y-1}} \approx \int_0^{y^*} \frac{dy}{e^{y/2}} = \frac{y^{3/2}}{3}$$

so we see the integral also converges at its lower limit $y \to 0$. 

![Graph showing $g_{3/2}(z)$ and $\zeta(3/2)$ as functions of $z$.]
So we conclude

\[ n = \frac{N}{V} = \frac{9^{3/2}(Z)}{2^3} \leq \frac{9^{3/2}(1)}{2^3} = \frac{2.612}{2.612} \left( \frac{2\pi mk_B T}{h^2} \right)^{3/2} \]

But we now have a contradiction!

For a system with fixed density of bosons \( n \), as \( T \) decreases we will eventually get to a temperature below which the above inequality is violated!

The temperature is

\[ T_0 = \left( \frac{m}{2\cdot16} \right)^{2/3} \frac{\hbar^2}{2\pi mk_B} \]

Solution to the paradox:

When we made the approx \( \frac{1}{V} \sum_k \rightarrow \int_{0}^{\infty} \frac{1}{(2\pi)^3} \int_0^\infty \text{d}k \, 4\pi k^2 \)

we gave a weight \( \frac{4\pi k^2}{(2\pi)^3} \) to states with wavevector \( \vec{k} \).

This gives zero weight to the state \( \vec{k} = 0 \), i.e. to the ground state. But as \( T \) decreases, more and more bosons will occupy the ground state, as it has the lowest energy. Thus when we approximate the sum by an integral, we should treat the ground state separately.

\[ \frac{1}{V} \sum_k n(\varepsilon(k)) \approx \frac{n(0)}{V} + \frac{1}{(2\pi)^3} \int_0^\infty \text{d}k \, 4\pi k^2 \, n(\varepsilon(k)) \]

ground state with occupation \( n(0) \).

This term is important when \( n(0)/V \) stays finite as \( V \to \infty \), i.e. a macroscopic fraction of bosons occupy the ground state.
Then we get

\[ M = \frac{V}{N} = n(0) + \frac{g_{3/2}(Z)}{\lambda^3} \]

\[ M = m_0 + \frac{g_{3/2}(Z)}{\lambda^3} \]

where \( m_0 = \frac{n(0)}{V} \) density of bosons in ground state

For a system with fixed \( m \), at higher \( T \) one can always choose \( Z \) so that \( m = \frac{g_{3/2}(Z)}{\lambda^3} \) and \( m_0 = 0 \).

But when \( T < T_c \) it is necessary to have \( m_0 > 0 \).

Using \( n(0) = \frac{Z}{1 - Z} \) we can write above as

\[ M = \frac{Z}{1 - Z} \left( \frac{V}{n} \right) + \frac{g_{3/2}(Z)}{\lambda^3} \]

For \( T > T_c \) we will have a solution to the above for some fixed \( Z < 1 \). In thermodynamic limit \( V \to \infty \), the first term will then vanish, i.e., the density of bosons in the ground state vanishes.

As \( T \to T_c \), \( Z \to 1 \) and the first term \( \left( \frac{Z}{1 - Z} \right) \left( \frac{1}{n} \right) \) diverges as \( Z \to 1 \) as \( V \to \infty \),

the needed density at \( T < T_c \):

\[ \frac{Z}{1 - Z} \left( \frac{1}{n} \right) = m_0 = m - \frac{g_{3/2}(1)}{\lambda^3} \]
To define the Bose-Einstein transition temperature below which the system develops a finite density of particles in the ground state $n_0$.

$n_0$ is also called the **condensate density**.

The particles in the ground state are called the condensate.

$$Z(T) \to 1 \text{ as } T \to T_c \quad \text{ and } \quad Z(T) = 1 \text{ for } T \leq T_c$$

$$\mu(T) \to 0 \quad \mu(T) = 0$$

For $T \leq T_c$,

$$n_0(T) = m - g_{3/2}(1) = m - 2.612 \left( \frac{2\pi m k_B T}{\hbar^2} \right)^{3/2}$$

$$n_0(T) = m \left( 1 - \left( \frac{T}{T_c} \right)^{3/2} \right)$$

The condensate density vanishes continuously as $T \to T_c$ from below.

At $T = 0$, all bosons are in the condensate.

At $T > T_c$, all bosons are in the "normal state".

At $0 < T < T_c$, a macroscopic fraction of bosons are in the condensate, while the remaining fraction are in the normal state, call it the "mixed state".
pressure - separate out ground state from sum as we
saw we needed to do in computing NV

\[ \frac{P}{k_b T} = \frac{1}{V} \ln \lambda = -\frac{1}{V} \sum_k \ln \left(1 - \frac{e^{-\beta E(k)}}{Z} \right) \]

\[ = -\frac{1}{V} \ln (1 - \frac{1}{Z}) - \frac{4 \pi}{(2 \pi)^3} \int_0^\infty dk^3 \ln \left(1 - \frac{e^{-\beta \hbar k^2 / 2 m}}{Z} \right) \]

\[ \uparrow \quad \text{ground state} \quad \uparrow \quad \text{all other } |k| > 0 \text{ states} \]

\[ = -\frac{1}{V} \ln \left(1 - \frac{1}{Z} \right) + \frac{9 \xi_2(Z)}{2^3} \frac{a}{Z} \quad a = \left( \frac{\hbar^2}{2 \pi m k_b T} \right)^{1/2} \]

where \( \xi_2(Z) = \frac{1}{\Gamma(\frac{3}{2})} \int_0^\infty dy \frac{y^{3/2}}{y^2 e^y - 1} \]

as derived when we began our discussion of
quantum gases

also recall the number of bosons occupying the ground state is

\[ n(0) = \frac{1}{Z^{-1} e^{\beta E(0)} - 1} = \frac{1}{Z^{-1} - 1} = \frac{Z}{1 - Z} \]

So \[ n(0) + 1 = \frac{Z}{1 - Z} + 1 = \frac{1}{1 - Z} \]

\[ \frac{P}{k_b T} = \frac{1}{V} \ln (n(0) + 1) + \frac{9 \xi_2(Z)}{2^3} \frac{a}{Z} \]

In the thermodynamic limit of \( V \to \infty \), the first term always
vanishes as \( n(0) \leq N = m V \) and
\[ \lim_{V \to \infty} \left[ \frac{\ln(mV)}{V} \right] = 0 \]

So the condensate does not contribute to the pressure.

This is not surprising as particles in the condensate have
\( \vec{k} = 0 \) and hence carry no momentum. In the kinetic theory
of gases, one sees that pressure arises from particles with
finite momentum \( |\vec{p}| > 0 \) hitting the walls of the container.
So \( \frac{\Phi}{k_B T} = \frac{g \xi/2(z)}{2^{5/2}} = g \xi/2(z) \left( \frac{2\pi m k_B T}{h^2} \right)^{3/2} \)

\[ \Phi = g \xi/2(z(T)) \left( \frac{2\pi m}{\hbar^2} \right)^{3/2} (k_B T)^{5/2} \]

- Equation of state
- For a system of fixed density \( m \), \( z \) must be chosen to be a function of \( T \) that gives the desired density \( m \).

Note: \( g \xi/2(z=1) = \xi/2 = 1.342 \)

In thermodynamic limit of \( V \to \infty \), \( z = 1 \) for \( T \leq T_c(m) \)

\[ \Rightarrow \Phi = g \xi/2(1) \left( \frac{2\pi m}{\hbar^2} \right)^{3/2} (k_B T)^{5/2} \quad \text{for} \quad T \leq T_c \]

- Critical temperature \( T_c \) depends on the system's fixed density

Note: \( \text{for} \ T \leq T_c \), the pressure \( p \propto T^{5/2} \) is independent of the system density!

\[ T_c(m) = \left( \frac{m^{2/3} \hbar^2}{2\pi m k_B} \right) \]

\[ T_c(m) = \left( \frac{m^{2/3} \hbar^2}{2\pi m k_B} \right) \]
Define \[ M_c(T) = 2.612 \left( \frac{2\pi m k_B T}{\hbar^2} \right)^{3/2} \text{inverse of } T_c(M) \]

\[ M_c(T) \] is the critical density at a given \( T \)

- a system with \( M > M_c(T) \) will be in a
  - Bose condensed mixed state at temperature \( T \).

phase diagram in \( p-T \) plane

\[ \phi \]

forbidden region above line \[ \frac{1}{2} \]

\( \propto \frac{1}{T} \) line

re-mixed state

on the line

normal state \( M \leq M_c(T) \)

below line

\[ T \]

Can also consider the transition in terms of \( p \) and \( v = \frac{v}{N} = \frac{1}{m} \) for various fixed \( T \).

At the transition \( \phi \propto T_c(m)^{5/2} \Rightarrow T_c(m) \propto m^{2/3} \)

\[ \Rightarrow \text{at the transition } \phi \propto (m^{2/3})^{5/2} = m^{5/3} = N^{-5/3} \]

below the transition \( p \) is independent of density \( m \) and hence independent of \( v \).

For fixed \( T \), the transition occurs when density \( M \)

exceeds \( M_c(T) \), or when \( v \) drops below \( \frac{1}{m_c(T)} \)

\[ v_c(T) \sim T^{-3/2} \]
Curves of $p$ vs $\nu$ at constant $T$

$\nu = \frac{3}{2} p$

$E = \frac{3}{2} p \nu = \frac{3}{2} p \nu = \frac{3}{2} k_B T \nu \frac{g_{5/2}(\nu)}{\nu}$

$z = 1$ in mixed state
$z < 1$ in normal state

In above we regard $E/N$ as a function of either $\nu$ or $z$. That is, we either determine $\nu$ for a given $z, T$ or we determine $z$ needed for a given $\nu, T$. (Recall $z = e^μ/N$,
$\nu = \frac{\nu}{N}$ ad $N$ and $\mu$ are conjugate variables)

Specific heat

$$\frac{C_v}{N k_B} = \left( \frac{\partial (E/N)}{\partial T} \right)_{\nu, N} = \frac{3}{2} \nu \int \frac{dT}{d^2} (T \frac{g_{5/2}(\nu)}{\nu}) + \frac{T}{\nu^3} \frac{\partial g_{5/2}(\nu)}{\partial \nu} \frac{d\nu}{dT}$$
For $T \leq T_c$, $z = 1$ so $\frac{d^2}{dT^2} = 0$ and only 1st term remains:

\[
\frac{T}{\lambda^3} \propto T^{5/2} \quad \text{so} \quad \frac{d}{dT} \left( \frac{T}{\lambda^3} \right) = \frac{5}{2} \left( \frac{T}{\lambda^3} \right) \frac{1}{T} = \frac{5}{2} \frac{1}{\lambda^3}
\]

\[z = 1 \text{ here for all } T \leq T_c\]

\[
\Rightarrow \frac{C_v}{Nk_B} = \frac{3}{2} \nu \left( \frac{5}{2} \frac{1}{\lambda^3} \right) g_{3/2}(1) = \frac{15}{4} g_{3/2}(1) \frac{\nu}{\lambda^3}
\]

\[
= \frac{15}{4} g_{3/2}(1) \nu \left( \frac{2\pi m k_B T}{\hbar^2} \right)^{3/2}
\]

Note, at $T_c$, $m = g_{3/2}(1)$, and $\nu = \frac{1}{m}$.

\[
\frac{C_v(T_c)}{Nk_B} = \frac{15}{4} \frac{g_{3/2}(1)}{g_{3/2}(1)} = \frac{15}{4} \frac{1.34}{2.612} = 1.925 \quad \text{this is larger than the classical ideal gas value of } 3/2.
\]

\[\text{So } \frac{C_v}{Nk_B} = 1.925 \left( \frac{T}{T_c} \right)^{3/2} \quad T \leq T_c\]

For $T > T_c$, $z$ varies with $T$ and we need to evaluate the 2nd term as well.

1st term: same as before.

2nd term: from lattice Appendix D Eq(10),

\[z \frac{d}{dz} \left[ g_v(z) \right] = g_{v+1}(z)\]

\[\Rightarrow \frac{d}{dz} \frac{g_{3/2}}{dT} = g_{3/2} \frac{1}{2} \frac{d^2}{dT^2}\]
To find $\frac{1}{2} \frac{d^2 z}{dT^2}$ consider our earlier result for the density when $T > T_C$:

$$n = \frac{9 \sqrt{2} (z)}{a^3}$$ determines $z(T)$ for fixed $n$.

For fixed $m$:

$$0 = \frac{d}{dt} \left( \frac{1}{a^3} \right) \frac{d}{dz} z^{3/2} + \frac{1}{a^3} \frac{d}{dz} z^{3/2} \frac{dz}{dt}$$

$$0 = \frac{3}{2} \frac{1}{a^3} T z^{3/2} + \frac{1}{a^3} \frac{d}{dz} z^{3/2} \frac{dz}{dt}$$

$$\Rightarrow \frac{3}{2} \frac{dz}{dt} = -\frac{3}{2} \frac{9 \sqrt{2} (z)}{z^{1/2}} \frac{1}{T}$$

$$\frac{C_v}{N k_B} = \frac{15}{4} \frac{9 \sqrt{2} (z)}{a^3} + \frac{3}{2} \frac{T}{a^3} \frac{9 \sqrt{2} (z)}{a^3} \left( -\frac{3}{2} \right) \frac{9 \sqrt{2} (z)}{a^3} \frac{1}{T}$$

Use $n = \frac{1}{a^3} \frac{d}{dz} \frac{9 \sqrt{2} (z)}{a^3} \Rightarrow \frac{T}{a^3} = \frac{1}{3} \frac{9 \sqrt{2} (z)}{a^3}$

$$\frac{C_v}{N k_B} = \frac{15}{4} \frac{9 \sqrt{2} (z)}{9 \sqrt{2} (z)} - \frac{9}{4} \frac{9 \sqrt{2} (z)}{9 \sqrt{2} (z)} \Rightarrow T > T_C$$

Note $\delta_{1/2} (1) = \frac{\sqrt{2}}{\sqrt{1}} \frac{1}{\sqrt{2}} \rightarrow \infty$

So as $T \rightarrow T_C^+$ from above, and $z \rightarrow 1$

$$\frac{C_v (T_C^+)}{N k_B} = \frac{15}{4} \frac{9 \sqrt{2} (1)}{9 \sqrt{2} (1)} - \frac{9}{4} \frac{9 \sqrt{2} (1)}{\infty} = \frac{15}{4} \frac{1.341}{1.925} \Rightarrow 2.612$$

$\Rightarrow [C_v$ is continuous at $T_C$]
Finally we want to show that although \( C_V \) is continuous at \( T_c \), \( \frac{dC_V}{dT} \) is discontinuous.

For \( T \leq T_c \)

\[
C_V = \frac{1.925}{Nk_B} (\frac{T}{T_c})^{3/2}
\]

\[
\frac{d}{dT} \left( \frac{C_V}{Nk_B} \right) = \frac{3}{2} \left( 1.925 \right) \left( \frac{T}{T_c} \right)^{1/2} \frac{1}{T_c} = 2.89 \left( \frac{T}{T_c} \right)^{1/2} \left( \frac{1}{T_c} \right)
\]

so slope at \( T_c^- \) (just below \( T_c \))

\[
\frac{d}{dT} \left( \frac{C_V}{Nk_B} \right) = \frac{2.89}{T_c} \quad \Rightarrow \quad T = T_c^-
\]

For \( T > T_c \)

\[
C_V = \frac{15}{4} g_{5/2} \left( \frac{z}{z_{1/2}} \right) - \frac{9}{4} \frac{\partial}{\partial T} \left( g_{3/2} \right)
\]

\[
\frac{d}{dT} \left( \frac{C_V}{Nk_B} \right) = \frac{15}{4} \frac{g_{3/2}}{g_{5/2}} \frac{dz}{dT} - \frac{g_{5/2}}{g_{3/2}} \frac{dz}{dT}
\]

\[
- \frac{9}{4} \frac{g_{3/2}}{g_{1/2}} \frac{dz}{dT} - \frac{g_{1/2}}{g_{3/2}} \frac{dz}{dT}
\]

\[
= \frac{1}{2} \frac{dz}{dT} \left\{ \frac{5}{4} \left( \frac{g_{3/2}}{g_{5/2}} - \frac{g_{3/2}}{g_{1/2}} \right) - \frac{9}{4} \left( \frac{g_{3/2}}{g_{5/2}} - \frac{g_{3/2}}{g_{1/2}} \right) \right\}
\]

\[
\text{Use } \frac{1}{2} \frac{dz}{dT} = -\frac{3}{2} \frac{g_{3/2}}{g_{5/2}} \frac{1}{T} \quad \text{as found earlier}
\]
\[
\frac{d}{dT} \left( \frac{CV}{Nk_B} \right) = -\frac{3}{8} \frac{g^{3/2}}{g^{1/2}} \left\{ 15 \left( 1 - \frac{g_{3/2}g_{1/2}}{g^{3/2}} \right)^2 - 9 \left( 1 - \frac{g_{3/2}g_{-1/2}}{g^{3/2}} \right)^2 \right\}
\]

Now as \( T \to T_c^+ \) from above, \( z \to 1 \), we have
\( g_{5/2}(1) \) and \( g_{3/2}(1) \) are finite, but \( g_{1/2}(1) \) and \( g_{-1/2}(1) \to \infty \)

\[\Rightarrow \text{at } T_c^+\]

\[
\frac{d}{dT} \left( \frac{CV}{Nk_B} \right) = \frac{45}{8T_c} \frac{g_{5/2}(1)}{g_{3/2}(1)} - \frac{27}{8T_c} \frac{g_{3/2}(1)}{g_{3/2}(1)} \frac{g_{-1/2}(1)}{g_{3/2}(1)}
\]

Now from Pathria Appendix D eq (8)

\[ g_{1/2}(1) = \lim_{a \to 0} \frac{P(1-V)}{a^{1-V}} \]

So

\[ \frac{g_{-1/2}(1)}{g_{3/2}(1)} = \lim_{a \to 0} \frac{P(3/2)}{a^{3/2}} \left( \frac{a^{1/2}}{P(1/2)} \right)^3 = \frac{P(3/2)}{\left[P(1/2)\right]^3} \]

\[= \frac{1}{2} \frac{\pi^{1/2}}{\pi^{3/2}} = \frac{1}{2\pi} \quad \text{since} \quad P(1/2) = \sqrt{\pi} \quad P(3/2) = \frac{1}{2} \sqrt{\pi} \]

\[
\frac{d}{dT} \left( \frac{CV}{Nk_B} \right) = \frac{45}{8} \frac{1.341}{2.612} \frac{1}{T_c} - \frac{27}{8} \frac{(2.612)^2}{2\pi} \frac{1}{T_c}
\]

\[= 2.89 - 3.66 = -0.77 \frac{1}{T_c} \]

\[\frac{d}{dT} \left( \frac{CV}{Nk_B} \right) = -0.77 \frac{1}{T_c} \quad T = T_c^+ \]

The slope of \( CV \) is discontinuous at \( T_c^+ \).
$C_v$ has a cusp at $T_c$

\[ \frac{C_v}{Nk_B} \]

\[ \frac{3}{2} \quad \text{as} \quad T \to \infty \]

\[ \frac{dC_v}{dT} > 0 \quad \text{for} \quad T = T_c^- \]

\[ \frac{dC_v}{dT} < 0 \quad \text{for} \quad T = T_c^+ \]

**Entropy**

For single species gas we had for Gibbs free energy

\[ G = N \mu \]

Also \[ G = E - TS + pV \] (since $G$ is Legendre transform of $E$ with respect to $S$ and $V$)

\[ \Rightarrow N \mu = E - TS + pV \]

\[ \Rightarrow S = \frac{E + pV - N\mu}{T} \]

\[ S = \frac{E + pV - \mu}{Nk_Bk_B T} \]

we had earlier \[ E = \frac{3}{2} pV \Rightarrow pV = \frac{2}{3} E \]

\[ \frac{S}{Nk_B} = \frac{5}{3} \frac{E}{k_B T} - \frac{\mu}{k_B T} \]
\[ z = e^{\frac{M}{k_B T}}, \quad z = 1 \quad \text{for} \quad T < T_c \]

We had earlier \[ \frac{E}{N} = \frac{3}{2} \frac{k_B T}{\lambda^3} g_{5/2}(z) \]

and \[ m = \frac{1}{v} = \frac{g_{3/2}(z)}{\lambda^3} \quad \text{for} \quad T > T_c \]

\[ \Rightarrow \quad \frac{S}{N k_B} = \frac{5}{2} \frac{v}{\lambda^3} g_{5/2}(z) - \ln z = \begin{cases} \frac{5}{2} \frac{g_{5/2}(z)}{g_{3/2}(z)} - \ln z & T > T_c \\ \frac{5}{2} \frac{v}{\lambda^3} g_{3/2}(1) & T \leq T_c \end{cases} \]

Note: For \( T < T_c \) we had that the density of the normal state \( n_0 = m - \frac{g_{3/2}(1)}{\lambda^3} m \) is the condensate, and a density \( \frac{g_{3/2}(1)}{\lambda^3} m \) in the normal state (i.e., the density of excited particles) \( \frac{g_{3/2}(1)}{\lambda^3} \equiv m_n \)

\[ \Rightarrow \quad \text{for} \quad T < T_c, \quad \frac{S}{N k_B} = \frac{5}{2} \left( \frac{m_n}{m} \right) \frac{g_{5/2}(1)}{g_{3/2}(1)} \quad \rightarrow 0 \quad \text{as} \quad T \rightarrow 0 \]

We can imagine that each normal particle carries

entropy \[ \frac{5}{2} k_B \frac{g_{5/2}(1)}{g_{3/2}(1)} \]

The entropy at \( T < T_c \)

is just the product above entropy per normal particle times the fraction of normal particles.

\[ \Rightarrow \quad \text{normal particles carry the entropy condensate has zero entropy} \]

Entropy difference per particle between normal state and condensed state \[ \frac{\Delta S}{N} = \frac{5}{2} k_B \frac{g_{5/2}(1)}{g_{3/2}(1)} \]
\[ L = T \Delta S = \frac{5}{2} k_B T \frac{g^{5/2}(1)}{g^{5/2}(1)} \]

energy released upon converting one normal particle to one condensate particle.

\[ \Rightarrow \text{mixed phase is like coexistence region of a 1st order phase transition (like water} \rightarrow \text{ice)} \]

\[ \Rightarrow \text{"two fluid" model of mixed region} \]