PHY 418 **Midterm Exam** **Spring 2018**

Please cross out any work you do not wish me to look at, and place a box around your final answer for each part.

1) [20 points]

Each quantity in the first row is equal to one quantity in the second row, via a Maxwell relation. Match the quantities in the first row to those in the second row, and write the correct second derivative of the appropriate thermodynamic potential that gives the Maxwell relation involved. When you specify the thermodynamic potential involved, be sure to write the variables that the potential depends on.

$$(a)\quad \left(\frac{\partial S}{\partial \mu}\right)_{T,V} \qquad (b)\ \left(\frac{\partial T}{\partial V}\right)_{S,N} \qquad (c)\quad \left(\frac{\partial \mu}{\partial T}\right)_{p,N}$$

$$(d)\ -\left(\frac{\partial p}{\partial S}\right)_{V,N} \qquad (e)\ \left(\frac{\partial N}{\partial T}\right)_{V,\mu} \qquad (f)\ -\left(\frac{\partial S}{\partial N}\right)_{T,p}$$

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2) [40 points]

Consider a classical ideal gas of N indistinguishable, non-interacting, particles confined to a region of three dimensional space by a harmonic potential $V(r)$. This might be a model for a gas of atoms in a magnetic trap. The single particle Hamiltonian is then,

$$\mathcal{H}^{(1)}(\mathbf{r},\mathbf{p}) = \frac{|\mathbf{p}|^2}{2m} + \frac{1}{2}m\omega_0^2|\mathbf{r}|^2.$$

a) [10 pts] Compute the average total energy E of the gas as a function of temperature T. Compute the total specific heat C.

b) [10 pts] Compute the density of particles $n(\mathbf{r})$ at position $\mathbf{r}$. Note, $n(\mathbf{r})$ should be normalized so that $\int d^3r\, n(\mathbf{r}) = N$.

c) [5 pts] What is the root mean square distance $\sqrt{\langle r^2\rangle}$ of particles from the origin?

d) [5 pts] What is the pressure of the gas $p(\mathbf{r})$ at a position $\mathbf{r}$?

e) [10 pts] What is the chemical potential of the gas $\mu(\mathbf{r})$ at position $\mathbf{r}$?

3) [40 points]

Consider a classical gas of N indistinguishable, non-relativistic, non-interacting atoms confined to a box of volume V in equilibrium at a temperature T. Each atom can be considered as a point particle, however it has an internal degree of freedom that can be in one of two possible states with energies $\epsilon_0 = 0$ and $\epsilon_1 > 0$ respectively.

a) [15 pts] Find the single particle partition function Q_1.

b) [10 pts] Find the chemical potential μ of the gas as a function of T, V and N.

c) [15 pts] Find the specific heat per particle at constant volume of the gas, $c_V = C_V/N$, and sketch it as a function of temperature.