PHY 418 **Final Exam** **Spring 2020**

This exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please do your best to write clearly. Please use a dark pen or pencil so that when you scan your solutions, they come out clearly. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at. Note, problems #1 and #2 are 20 points each, while problems #3 and #4 are 30 points each.

You have three hours to work on the exam, and an additional 45 min to scan or photograph your work and email it back to me at: stte@pas.rochester.edu The exam begins at 12:30pm and ends at 3:30pm. You then have until 4:15pm to email me your work.

Please write the academic honesty pledge, and sign your name, at the top of your work: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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1) [20 points total]

Consider a classical gas of N identical, non-relativistic, non-interacting atoms confined to a box of volume V in equilibrium at a temperature T. Each atom can be considered as a point particle, however it has an internal degree of freedom that can be in one of two possible states with energies $\epsilon_0 = 0$ and $\epsilon_1 > 0$ respectively.

a) [10 pts] Find the chemical potential μ of the gas as a function of T, V and N.

b) [10 pts] Find the specific heat per particle at constant volume c_V of the gas, and sketch it as a function of temperature (do not forget the sketch!).

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2) [20 points total]

Consider a point particle of mass m attached to a harmonic spring with spring constant $\kappa = m\omega_0^2$. Consider that the particle moves only in one dimension, labeled by the coordinate x, and the position of the particle at rest is at $x = 0$. The particle is in equilibrium at a temperature T.

a) [8 pts] Assume that the particle behaves clasically. What is the root-mean-squared fluctuation $\Delta x \equiv \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ of the particle about its equilibrium position $\langle x\rangle$?

b) [8 pts] Assume that the particle must be treated quantum mechanically. What now is Δx? (Hint: recall that for the quantum harmonic oscillator, the expected value of the kinetic energy equals the expected value of the potential energy - this is the quantum virial theorem.)

c) [4 pts] Show that your answer in (b) reduces to your answer in (a) in the appropriate limit.

(problems #3 and #4 are on the back side)

3) [30 points]

Consider an ideal gas of N non-interacting, extremely relativistic, spin 1/2 fermions, whose energy-momentum relationship is well approximated by $\epsilon(\mathbf{p}) = c|\mathbf{p}|$. The particles are confined to a cubic three dimensional box of fixed volume $V = L^3$. The density of particles in the gas is $n = N/V$, and we consider the thermodynamic limit of $V \to \infty$ with n held constant.

a) [10 pts] Find the density of single particle states per unit energy per unit volume, $g(\epsilon)$, for the gas.

b) [10 pts] Find the Fermi energy of the gas.

c) [10 pts] Compute the pressure of the gas at $T = 0$.

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4) [30 points]

In Problem 3 of Problem Set 6 you showed that, in the grand canonical ensemble, the fluctuation in the total number of particles in an ideal (i.e. non-interacting) quantum gas was,

$$\langle N^2\rangle - \langle N\rangle^2 = \sum_i \left[\langle n_i^2\rangle - \langle n_i\rangle^2\right] = \sum_i \langle n_i\rangle(1 \pm \langle n_i\rangle)$$

where n_i is the occupation number of particles in single particle eigenstate i with energy ϵ_i. In the right-hand term above, the $+$ sign is for a gas of bosons, while the $-$ sign is for a gas of fermions.

For free particles in a three dimensional cubic box of side length L, in equilibrium at temperature T, compute the relative fluctuation in the number of particles, $\frac{\sqrt{\langle N^2\rangle - \langle N\rangle^2}}{\langle N\rangle}$, and show how it behaves to leading order as $\langle N\rangle \to \infty$, as a function of T,

a) [15 pts] for a gas of fermions in the degenerate limit (i.e., $T \ll T_F$). How does this compare to what you would expect for a classical gas of particles?

b) [15 pts] for a gas of bosons at a temperature below the Bose-Einstein condensation temperature ($T < T_c$). You may wish to focus in particular on the behavior of the particles in the ground state. What is strange about your result?

Note, you may not need to do exact analytical calculations to get all the details; just make sound arguments to support your conclusions!