PHY 418 **Midterm Exam** **Spring 2020**

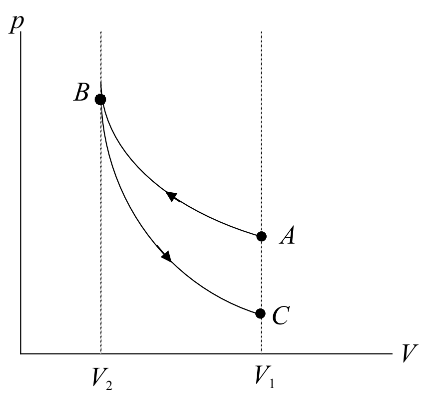
This exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please do your best to write clearly. Please use a dark pen or pencil so that when you scan your solutions, they come out clearly. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at. Note, each problem is worth a different number of points.

You have two hours to work on the exam, and an additional 45 min to scan or photograph your work and email it back to me at: stte@pas.rochester.edu The exam begins at 10:30am, and all work must be back to me by 1:15pm.

Please write the academic honesty pledge, and sign your name, at the top of your work: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

1) [25 points]

Consider a classical ideal gas of N point particles, that is compressed and then expanded, as shown in the diagram below. The gas starts at point A in the $p-V$ diagram, with volume V_1 at temperature T_1. It is then *isothermally* compressed (i.e. at constant temperature) to volume V_2, denoted point B on the diagram. The gas is then *adiabatically* expanded (i.e. at constant entropy) back to volume V_1, denoted point C on the diagram. What is the final temperature T_2 at point C? Express your answer in terms of T_1, V_1, and V_2.



It might help you to know that the entropy S of an ideal gas is

$$S(E,V,N) = \left(\frac{N}{N_0}\right) S_0 + Nk_B \ln\left[\left(\frac{E}{E_0}\right)^{3/2}\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-5/2}\right]$$

where E, V, N are the total energy, total volume, and total number of particles, and E_0, V_0, N_0, S_0 are constants.

(problems 2 and 3 on back of page)

2) [45 points]

Consider a cylindrical column of cross-sectional area $\mathcal{A}$ in the xy plane, and infinite height along the z axis, i.e., $0 \leq z < \infty$. The column is filled with classical ideal gas of N non-interacting, non-relativistic, indistinguishable point particles of mass m. The gas is in equilibrium at a fixed temperature T. The force of gravity acts upon the particles of gas in the column via the gravitational potential energy $U(z) = mgz$, where z is the height within the column, and the constant g is the gravitational acceleration at the surface of the Earth.

a) [15 pts] What is the density of the gas $n(z)$ at height z within the column?

b) [15 pts] What is the pressure of the gas $p(z)$ at height z within the column?

c) [15 pts] What is the chemical potential of the gas $\mu(z)$ at height z within the column?

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3) [30 points]

Consider a classical gas of indistinguishable particles in equilibrium in the grand canonical ensemble. Label the set of allowed states as $\{i\}$, where state i has total energy E_i and total number of particles N_i. You can think of state i as representing some small finite cell of classical phase space. The grand canonical partition function can then be written as

$$\mathcal{L} = \sum_i \mathrm{e}^{-\beta(E_i - \mu N_i)} = \sum_i z^{N_i} \mathrm{e}^{-\beta E_i}$$

where $\beta = 1/k_B T$, μ is the chemical potential, and $z = \mathrm{e}^{\beta\mu}$ is the fugacity.

a) [5 pts] Write an expression that gives the probability $\mathcal{P}_i$ for the system to be found in a particular state i.

b) [6 pts] What is the probability $\mathcal{P}(N)$ that the system has a given number of particles N? Express your answer in terms of $\mathcal{L}$, z and the N-particle canonical partition function Q_N.

c) [7 pts] Find general expressions for the average $\langle N \rangle$, and variance $\langle N^2 \rangle - \langle N \rangle^2$, of the number of particles in terms of appropriate derivatives of $\ln \mathcal{L}$.

Now suppose that the particles are non-interacting.

d) [7 pts] Show that the probability $\mathcal{P}(N)$ for the system to have N particles has the form,

$$\mathcal{P}(N) = \frac{\lambda^N}{N!} \mathrm{e}^{-\lambda}$$

and express λ in terms of z and the single-particle partition function Q_1.

e) [5 pts] Find $\langle N \rangle$ and $\langle N^2 \rangle - \langle N \rangle^2$ in terms of z and Q_1.