PHYS 418 **Endterm Exam** **Spring 2023**

For this exam you may use one 8.5"x11" sheet of paper on which you have written whatever notes you wish on both sides of the page. Except for that sheet, this exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please write clearly. Please use a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work:
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

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1) [35 points total]

Consider atoms A that can bind together to form a diatomic molecule A_2,

$$2A \quad \leftrightarrow \quad A_2$$

The binding energy of the molecule is Δ (i.e. the ground state of the molecule A_2 has energy $-\Delta$ compared to the ground state of two unbound atoms). The mass of A_2 is just twice the mass of A. Assume that the atoms A and diatomic molecules A_2 can be treated as non-interacting, non-relativistic, indistinguishable, classical point particles (i.e. ignore any rotational, vibrational, or electronic excitations). The system is contained in a cubic box of volume V at a fixed temperature T.

a) [5 pts] If one denotes the chemical potential of the isolated atoms A as μ_A, and the chemical potential of the molecules A_2 as μ_B, what is the relation between μ_A and μ_B when the system is in chemical equilibrium? What is the relationship between the fugacities z_A and z_B?

b) [10 pts] Let N_A be the number of atoms A, and N_B be the number of molecules A_2. Express the fugacity z_A in terms of the density of atoms $n_A = N_A/V$ and the temperature T. Express the fugacity z_B in terms of the density of molecules $n_B = N_B/V$ and T.

c) [15 pts] Suppose that there are initially N atoms A and no molecules A_2 in the box. What will be the ratio N_A/N_B of the number of atoms A to the number of molecules A_2 when the system comes into chemical equilibrium?

d) [5 pts] What can you say about N_A/N_B when $\Delta \ll k_BT$? when $\Delta \gg k_BT$?

(turn over for problems 2 and 3)

2) [30 points total]

Consider an ideal gas of N indistinguishable, non-interacting, extremely relativistic, spin $\frac{1}{2}$ fermions, whose energy-momentum relationship is well approximated by $\varepsilon(\mathbf{p}) = c|\mathbf{p}|$. The particles are confined to a cubic three dimensional box of fixed volume $V = L^3$. The density of particles in the gas is $n = N/V$, and we consider the thermodynamic limit of $V \to \infty$ with n held constant.

a) [10 pts] Find the density of single particle states per unit energy per unit volume, $g(\varepsilon)$, for the gas.

b) [10 pts] Find the Fermi energy of the gas.

c) [10 pts] Compute the pressure of the gas at $T = 0$.

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3) [35 points total]

Consider a two-dimensional gas of N indistinguishable, non-interacting, non-relativistic, spin-zero bosons of mass m in an external harmonic potential $U(\mathbf{r}) = \frac{1}{2}m\omega_0^2|\mathbf{r}|^2$, where $\mathbf{r} = (x, y)$. This might be taken as a model for bosons in a magnetic trap. The quantized single particle energy levels are given by $\varepsilon(n_x, n_y) = \hbar\omega_0(n_x + n_y + 1)$, where $n_x, n_y = 0, 1, 2, \ldots$ are integers.

a) [10 pts] Compute the density of states per unit energy, $g(\varepsilon)$. The density of states is such that $g(\varepsilon)d\varepsilon$ is the number of single particle states between ε and $\varepsilon + d\varepsilon$. You may assume that the thermal energy is much greater than the spacing between the energy levels, i.e. $k_B T \gg \hbar\omega_0$.

b) [5 pts] What is the largest value that the fugacity z can have?

c) [10 pts] Show that one does have Bose-Einstein condensation at sufficiently low temperature.

d) [5 pts] Find the Bose-Einstein condensation temperature T_c.

e) [5 pts] Find how the number of particles in the condensate $N_0(T)$ (i.e. in the ground state) varies with temperature for $\hbar\omega_0/k_B \ll T \leq T_c$.