PHY 418 **Midterm Exam** **Spring 2023**

For this exam you may use one 8.5"x11" sheet of paper on which you have written whatever notes you wish. Except for that sheet, this exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please write clearly. Please use a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work:
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

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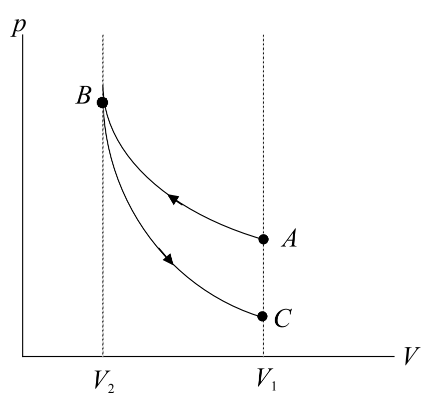
1) [30 points total]

a) [15 pts] Consider the thermodynamic derivatives below. For each case of the five cases, use the Maxwell relations to find another thermodynamic derivative to which the given one is equal. Make sure to get your signs correct!

$$(i)\ \left(\frac{\partial V}{\partial T}\right)_{p,N} \qquad (ii)\ \left(\frac{\partial T}{\partial V}\right)_{S,N} \qquad (iii)\ \left(\frac{\partial p}{\partial \mu}\right)_{T,V} \qquad (iv)\ \left(\frac{\partial \mu}{\partial p}\right)_{S,N} \qquad (v)\ \left(\frac{\partial \mu}{\partial T}\right)_{V,N}$$

b) [15 pts] Consider a classical ideal gas of N particles, that is compressed and then expanded, as shown in the diagram below. The gas starts at point A in the p-V diagram, with volume V_1 at temperature T_1. It is then *isothermally* compressed (i.e. at constant temperature) to volume V_2, denoted as point B on the diagram. The gas is then *adiabatically* expanded (i.e. at constant entropy) back to volume V_1, denoted point C on the diagram. What is the final temperature T_2 at point C? Express your answer in terms of T_1, V_1, and V_2.



It might help you to know that the entropy S of an ideal gas has the form,

$$S(E,V,N) = \left(\frac{N}{N_0}\right) S_0 + N k_B \ln\left[\left(\frac{E}{E_0}\right)^{3/2}\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-5/2}\right]$$

where E, V, N are the total energy, total volume, and total number of particles, and E_0, V_0, N_0, and S_0 are constants.

2) [30 points total]

Consider a classical ideal gas of N non-interacting, non-relativistic, indistinguishable particles of mass m, confined to a one dimensional box of length L, in equilibrium at temperature T. Inside the box the particles feel a potential energy $U(x)$.

a) [15 pts] What is the probability density $\mathcal{P}(x)$ that a particle will be found at position x?

b) [15 pts] Suppose the potential energy has the form: $U(x) = \begin{cases} 0 & \text{for} \quad 0 \leq x < L_1 \\ U_0 & \text{for} \quad L_1 \leq x \leq L \end{cases}$

What is the total probability that a particle will be found to have a position x such that x is in the range $L_1 \leq x \leq L$?

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3) [40 points total]

Consider a classical ideal gas of N non-interacting, non-relativistic, indistinguishable particles of mass m, confined to a box of volume V, in equilibrium at a temperature T. Each particle i has an internal degree of freedom ε_i that can have two possible energy values, 0 or δ, where $\delta > 0$. The Hamiltonian for the gas is therefore,

$$\mathcal{H} = \sum_{i=1}^{N} \left[\frac{|\mathbf{p}_i|^2}{2m} + \varepsilon_i \right] \qquad \text{where } \mathbf{p}_i \text{ is the momentum of particle } i$$

a) [10 pts] Find the canonical partition function $Q_N(T, V)$ for this gas.

b) [7 pts] Find the pressure p as a function of T, V and N.

c) [7 pts] Find the chemical potential μ as a function of T, V, and N.

d) [8 pts] Find the specific hear per particle at constant volume of the gas, $c_V = C_V/N$, and sketch it as a function of temperature.

e) [8 pts] Let n be the number of particles for which the internal degree of freedom is in the state with $\varepsilon_i = \delta$. What is the average number $\langle n \rangle$, and what is the variance of this number, $\langle n^2 \rangle - \langle n \rangle^2$?