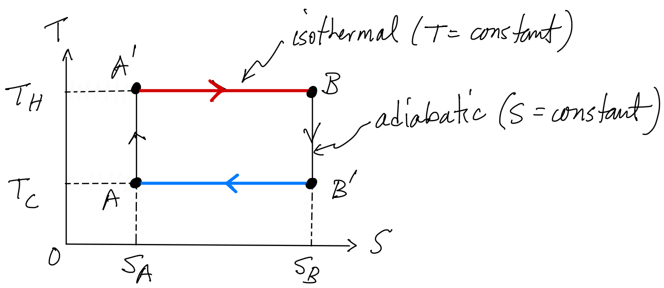
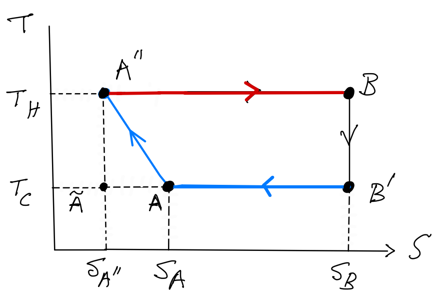
Unit 1-10.S: Heat Engines - Supplement

Recall the Carnot Cycle, shown below. Heat is transferred from the hot reservoir to the working body when the system moves along the segment of the cycle denoted in red, and heat is transferred from the working body to the cold reservoir when the system moves along the segment of the cycle denoted in blue.



One student asked why it was not possible to improve the efficiency of the cycle, by changing the cycle as shown below, so that there is a greater area enclosed by the cycle loop.



At one point I thought that perhaps the curve from A to A'' in this new cycle was not physically possible. It is a curve on which the temperature T increases, even though heat $\Delta Q = T\Delta S < 0$ is being extracted from the system. This segment of the curve has $dS/dT < 0$ and that looks like a specific heat, which by stability analysis is always positive.

However this argument is not correct. The dS/dT on this segment of the curve is not simply related to the specific heats at constant pressure C_p and at constant volume C_V that must be positive, since p and V are varying along this curve. And since $dE = TdS - pdV$, it is indeed possible for the internal energy (and hence the temperature) to increase, while the entropy decreases (heat is extracted), if sufficient mechanical work is done *on* the system ($pdV < 0$) for example by compressing a piston.

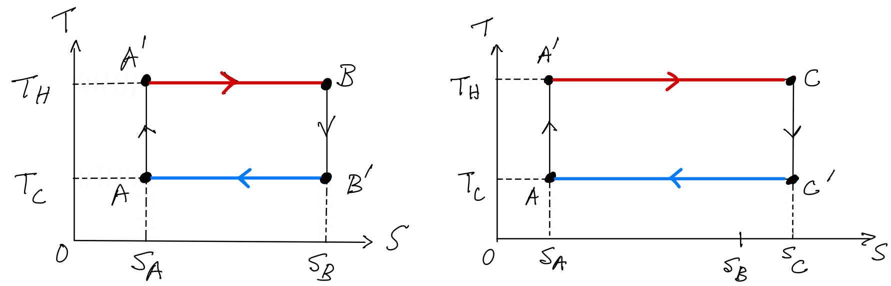
The point is that the above modification *reduces* the efficiency of the cycle. The efficiency is given by,

$$\varepsilon = 1 - \frac{Q_C}{Q_H} \tag{1.10.S.1}$$

where Q_H is the heat absorbed from the hot reservoir, and Q_C is the heat released to the cold reservoir. In the second cycle shown above, Q_H is the area under the red curve going from A'' to B, so $Q_H = T_H(S_B - S_{A''})$. Q_C is (-) the area under the blue curve going from B' to A to A'', so $Q_C = T_C(S_B - S_{A''}) + \tilde{Q}$, where $\tilde{Q}$ is the area of the triangle going from A to $\tilde{A}$ to A'' and back to A. The efficiency of this cycle is then,

$$\varepsilon = 1 - \frac{T_C(S_B - S_{A''}) + \tilde{Q}}{T_H(S_B - S_{A''})} = 1 - \frac{T_C}{T_H} - \frac{\tilde{Q}}{T_H(S_B - S_{A''})} < 1 - \frac{T_C}{T_H} \tag{1.10.S.2}$$

So the efficiency is not in general determined just by the area enclosed by the loop describing the cycle. To see that this is so, consider the two different Carnot cycles sketched below.



Clearly the area bounded by the loop on the left, from A to A' to B to B' and back to A, is less than the area bounded by the loop on the right, from A to A' to C to C' and back to A. Yet the efficiency of the two loops is the same, $\varepsilon = 1 - T_C/T_H$.

If the efficiency of the two cycles above is the same, then what is the difference between them? The difference is a matter of capacity. The cycle on the right absorbs more heat from the hot reservoir and releases more heat to the cold reservoir. Since we also have $\varepsilon = W/Q_H$, the cycle on the right also does more mechanical work W than does the loop on the left.