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RKKY Interaction and Spin Glasses

Mn-Cu alloys for example

In our discussion of the Lindhard dielectric function we saw that:

If there is a potential energy $U(\vec{r})$ that couples to the electron density, ie the perturbation in the Hamiltonian is

$$\sum_i U(\vec{r}_i) = \int d^3r\, U(\vec{r})\, n(\vec{r})$$

with $n(\vec{r}) \equiv \sum_i \delta(\vec{r}-\vec{r}_i)$ is the electron density then U induces a change in electron density $\delta n(\vec{r})$ given, in Fourier transform space, by

$$\delta n(\vec{q}) = \chi(\vec{q})\, U(\vec{q})$$

with $$\chi(\vec{q}) \equiv \frac{2}{V} \sum_k \frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k}$$

where f_k is the Fermi occupation function for the free electron state with wave vector $\vec{k}$ and energy ε_k.

A spin glass is a material where the magnetic moments on randomly positioned magnetic impurities seem to freeze into random orientations as the temperature cools down. Here we discuss the RKKY interaction between such impurity spins and show how it can lead to a spin glass.

Consider a magnetic impurity with spin $\vec{S}_0$ located at position $\vec{R}_0$. We will assume the interaction of $\vec{S}_0$ with the conduction electrons is via a <u>local</u> spin-spin interaction.

$$\delta H = -J\mu_B \vec{S}_0 \cdot \sum_i \underbrace{\vec{s}_i}_{\text{spin of electron } i} \underbrace{|\psi_i(\vec{R}_0)|^2}_{\text{probability for electron } i \text{ to be at position } \vec{R}_0}$$

$$= J \vec{S}_0 \cdot \underbrace{\vec{m}(R_0)}_{\text{magnetization density of electrons}}$$

Let us take the direction of $\vec{S}_0$ to be $\hat{z}$. Then

$$\delta H = -J\mu_B S_0 \left[\underbrace{n_\uparrow(\vec{R}_0)}_{\text{density of } \uparrow \text{ electrons}} - \underbrace{n_\downarrow(\vec{R}_0)}_{\text{density of } \downarrow \text{ electrons}} \right]$$

$$= \delta H_\uparrow + \delta H_\downarrow$$

$$\delta H_\uparrow \equiv -J\mu_B S_0 \, n_\uparrow(\vec{R}_0) = \int d^3r \; U_\uparrow(\vec{r}) \, n_\uparrow(\vec{r})$$

$$\delta H_\downarrow = +J\mu_B S_0 \, n_\downarrow(\vec{R}_0) = \int d^3r \; U_\downarrow(\vec{r}) \, n_\downarrow(\vec{r})$$

$$\text{where} \begin{cases} U_\uparrow(\vec{r}) = -J\mu_B S_0 \, \delta(\vec{r}-\vec{R}_0) \\ U_\downarrow(\vec{r}) = J\mu_B S_0 \, \delta(\vec{r}-\vec{R}_0) \end{cases}$$

We then have that $U_\uparrow$ and $U_\downarrow$ induce perturbations $\delta n_\uparrow$ and $\delta n_\downarrow$ in the spin ↑ and spin ↓ electron densities.

$$\delta n_\uparrow(\vec{q}) = \tfrac{1}{2}\chi(\vec{q})\, U_\uparrow(\vec{q})$$

$$\delta n_\downarrow(\vec{q}) = \tfrac{1}{2}\chi(\vec{q})\, U_\downarrow(\vec{q}) = -\tfrac{1}{2}\chi(\vec{q})\, U_\uparrow(\vec{q})$$

factor of $\frac{1}{2}$ since $n_\uparrow$ and $n_\downarrow$ are both $\frac{1}{2}$ of total density $n = n_\uparrow + n_\downarrow$

induced electron magnetization is then

$$m_z(\vec{q}) = -\mu_B\left[\delta n_\uparrow(\vec{r}) - \delta n_\downarrow(\vec{r})\right]$$
$$= -\mu_B\,\chi(\vec{q})\, U_\uparrow(\vec{q})$$

Now $U_\uparrow(\vec{r}) = -J\mu_B S_0\,\delta(\vec{r}-\vec{R}_0)$

so $U_\uparrow(\vec{q}) = \int d^3\vec{r}\, e^{-i\vec{q}\cdot\vec{r}}\, U_\uparrow(\vec{r})$

$$= -J\mu_B S_0\, e^{-i\vec{q}\cdot\vec{R}_0}$$

$$\Rightarrow\quad m_z(\vec{r}) = \int\frac{d^3q}{(2\pi)^3}\, J\mu_B^2 S_0\, \chi_q\, e^{-i\vec{q}\cdot\vec{R}_0}\, e^{i\vec{q}\cdot\vec{r}}$$

$$= J\mu_B^2 S_0 \int\frac{d^3q}{(2\pi)^3}\,\chi(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}-\vec{R}_0)}$$

$$m_z(\vec{r}) = J\mu_B^2 S_0\, \chi(\vec{r}-\vec{R}_0)$$

Fourier transform of $\chi(\vec{q})$ evaluated at position $\vec{r}-\vec{R}_0$

induced magnetization is in the same direction as $\vec{S}_0$, so

$$\vec{m}(\vec{r}) = +J\mu_B^2\, \vec{S}_0\, \chi(\vec{r}-\vec{R}_0)$$

For many impurities $\vec{S}_i$ at positions $\vec{R}_i$, the total induced electron magnetization is obtained from the above by superposition

$$\vec{m}(\vec{r}) = +J\mu_B^2 \sum_i \vec{S}_i\, \chi(\vec{r}-\vec{R}_i)$$

The interaction Hamiltonian is then

$$\delta H = J \sum_j \vec{S}_j \cdot \vec{m}(\vec{R}_j)$$

$$\boxed{\delta H = +J^2\mu_B^2 \sum_{i,j} \vec{S}_j \cdot \vec{S}_i\, \chi(\vec{R}_j-\vec{R}_i)}$$

Above result shows how the magnetization of the conduction electrons mediates an interaction between the two magnetic impurities $\vec{S}_i$ and $\vec{S}_j$. If $\chi(\vec{R}_j-\vec{R}_i) < 0$ then the interaction is <u>ferromagnetic</u>. If $\chi(\vec{R}_j-\vec{R}_i) > 0$ then the interaction is anti<u>ferromagnetic</u>.

Now $$\chi(\vec{r}) = \int \frac{d^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\, \chi(\vec{q})$$

with $$\chi(\vec{q}) = 2\int \frac{d^3k}{(2\pi)^3} \left[\frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k}\right]$$

$$= -g(\varepsilon_F)\left[1 + \frac{1-x^2}{2x} \ln\left|\frac{1+x}{1-x}\right|\right]$$

where $x = q/2k_F$

As discussed in connection with the Lindhard dielectric function, $\chi(\vec{q})$ has a singularity at $x=0$ or $|\vec{q}| = 2k_F$. This results in $\chi(\vec{r})$ having a piece that goes as

$$\chi(\vec{r}) \sim \frac{1}{r^3}\cos(2k_F r)$$

which oscillates in sign depending on the value of the distance r. Since the magnetic impurities $\vec{S}_i$ are randomly positioned in the metal, with an average spacing several times the atomic lattice constant, then $k_F|\vec{R}_i - \vec{R}_j|$ in general is large and hence $\chi(\vec{R}_i - \vec{R}_j)$ will be randomly positive or negative, according to the particular random separation between the spins. Thus the interaction between spins $\vec{S}_i$ and $\vec{S}_j$ is randomly ferro or anti-ferro magnetic. This is the the model interaction for a "spinglass" where the spins freeze into random orientations as T decreases