Hall coefficient is

$$R \equiv -\frac{\rho_{xy}}{H} \quad \text{(see Quantum Hall effect notes)}$$

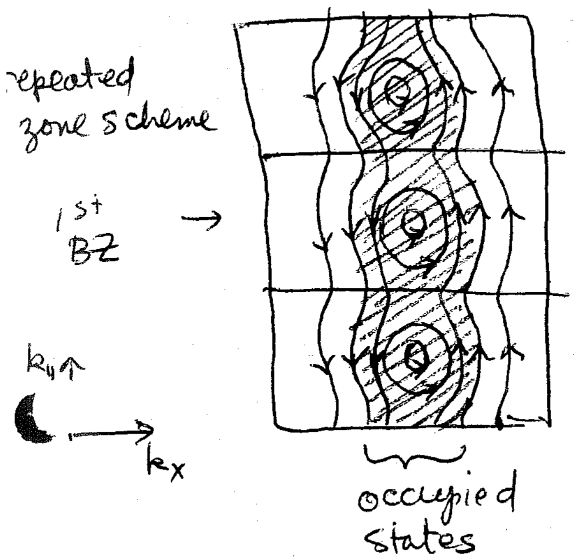
$$= -\frac{\omega_c \tau}{\sigma_0 H} = -\frac{eH}{m^* c}\,\frac{\tau m^*}{n e^2 \tau H} = -\frac{1}{nec} \quad \text{as before}$$

magnetoresistance

$$\rho_{xx} = \rho_{xy} = \frac{1}{\sigma_0}$$ saturates to finite value as $H \to 0$ just as was found in Drude model, except now m is m_{eff} & there are several partially filled bands.

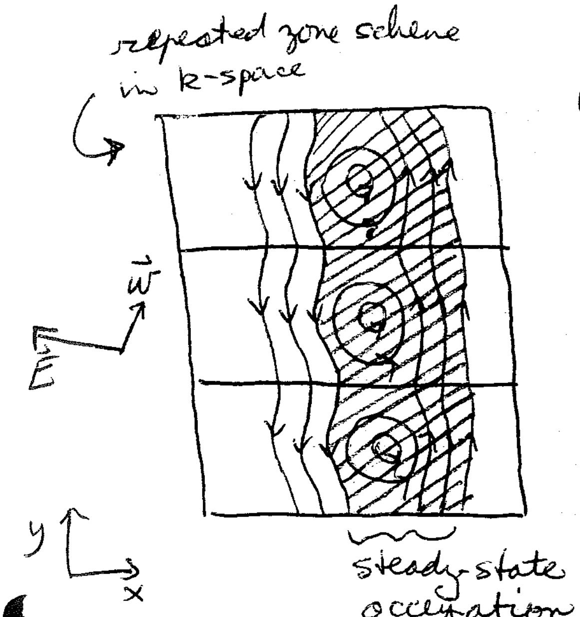
Case (2) Neither all occupied states, nor all unoccupied states have closed orbits $\Rightarrow$ in either electron or hole picture there are open orbits we have to consider

Now we will find that the $\langle \dot{\vec{k}} \rangle$ contribution to current $\vec{j}$ from these open orbits no longer vanishes in the $\omega_c \tau \to \infty$ limit, and it dominates over the drift contribution to the current $-ne\vec{w}$.



when $\vec{E}=0$, $\vec{H} = H\hat{z}$ induces motion in orbits on the constant energy surfaces. An electron moving in an open orbit in $\vec{k}$-space in the $+\hat{k}_y$ direction, gives a particle current in real space in the $+\hat{x}$ direction (rotate clockwise by $90°$ about $\hat{H}$). However when $\vec{E}=0$, each occupied open orbit going in one direction is paired with an occupied open orbit going in the opposite direction, so the net current is zero.

[Note: For an open orbit traveling along $\hat{k}_y$, $k_y(t)$ is periodic in time $\rightarrow$ $v_y = \left\langle \frac{\partial \mathcal{E}}{\hbar \partial k_y} \right\rangle = 0$ averaged over time. But $k_x(t) \approx$ constant + oscillation $\Rightarrow$ $v_x = \left\langle \frac{\partial \mathcal{E}}{\hbar \partial k_x} \right\rangle \neq 0$ $\Rightarrow$ electron moves in $\hat{x}$ direction.]



when $\vec{E} \neq 0$, in steady state, there will be an imbalance in occupation of open orbits, so that those orbits which ~~of~~ absorb energy from the E-field have a larger population than those which lose energy to the field. ($\vec{E}$ field heats up metal!)

Open orbits in $+\hat{k}_y$ direction have real space direction $+\hat{x}$ $\Rightarrow$ they gain energy from E field if $E_x < 0$ as energy absorbed $\underbrace{\text{is}}_{\text{between collisions}}$ $-e\vec{E}\cdot\vec{v}\,\tau > 0$.

Open orbits in $-\hat{k}_y$ directions have real space direction $-\hat{x}$ $\Rightarrow$ they lose energy if $E_x < 0$.

$E_x < 0 \Rightarrow$ net

$v_x > 0 \Rightarrow j_x < 0$

so $j_x \sim E_x$ to lowest order in E

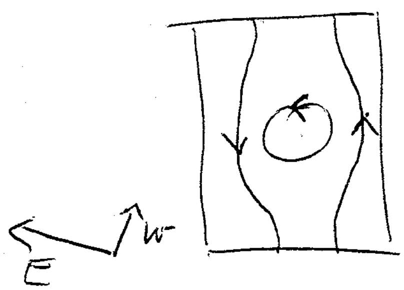
$\vec{j} \sim \hat{x}(\vec{E}\cdot\hat{x})$

$\Rightarrow$ ~~We assume therefore that the imbalance in occupation of open orbits in steady state ~~goes like~~ gives rise to a net current. If $\hat{n}$ is the direction in real space of the open orbits, then this contribution to current $\vec{j}$ is in the $\hat{n}$ direction, and proportional to some function of $\vec{E}\cdot\hat{n}$.~~

$\Rightarrow \vec{j}_{\text{open orbits}} \sim \hat{n}\, g(\vec{E}\cdot\hat{n})$ — expand in small $\vec{E}$, Equivalently, since $\bar{\mathcal{E}} = \mathcal{E} - \hbar \vec{k}\cdot\vec{w}$ is conserved between collisions, if $\Delta\mathcal{E} = -e\vec{E}\cdot\vec{v}\,\tau$ is energy absorbed by electron from E-field then

$$\Delta\bar{\mathcal{E}} = 0 \Rightarrow \Delta\mathcal{E} = \hbar\vec{w}\cdot\Delta\vec{k}$$

So again we see in our example



that ~~it~~ it is the ~~left~~ right hand open orbits moving along $+\hat{k}_y$ that absorb energy, ie $\vec{w}\cdot\Delta\vec{k}>0$ for these orbits, while $\vec{w}\cdot\Delta\vec{k}<0$ for left hand open orbits moving along $-\hat{k}_y$.

~~right hand open orbits absorb energy from field~~
~~left hand open orbits lose energy to field~~ ~~$\Rightarrow$ right hand orbits~~

So both $\vec{w}\cdot\Delta\vec{k}$ and $-E\cdot v$ tell how much energy the electron absorbs from E-field

This imbalance in steady state occupation of open orbits is determined by the quantity $-e\vec{E}\cdot\vec{v}\tau$, the energy absorbed by electron from $\vec{E}$-field in between collisions.
If $\hat{n}$ is real space direction of open orbit, $\Rightarrow \langle\vec{v}\rangle$ is in $\hat{n}$ direction, so the current due to open orbits is in the $\hat{n}$ direction, and is some function of $(\vec{E}\cdot\hat{n})$.

$$\vec{j}_{\substack{open\\orbits}} = \hat{n}\, g(\vec{E}\cdot\hat{n})$$

– expand for small $\vec{E}$, using $j=0$ when $\vec{E}=0$, and $j(E) = -j(-E)$

$$\vec{j}_{\substack{open\\orbits}} \sim \hat{n}\,(\hat{n}\cdot\vec{E})$$

where proportionality constant is independent of magnetic field H

We can write the contribution to conductivity tensor due to open orbits as

$$\vec{j}_{\substack{open\\orbits}} = \tilde{\sigma}\cdot\vec{E} \qquad \text{where } \tilde{\sigma} = \lambda\sigma_0\,\hat{n}\,\hat{n}$$

λ constant indep of H

If we choose $\hat{n}$ in $\hat{x}$ direction

$$\tilde{\sigma} = \lambda\sigma_0\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

If we treat the contribution to conductivity tensor from closed orbits as before, we get for total conductivity tensor

$$\overleftrightarrow{\sigma} = \frac{\sigma_0}{(\omega_c\tau)^2}\begin{pmatrix}1 & -\omega_c\tau\\ \omega_c\tau & 1\end{pmatrix} + \lambda\sigma_0\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

$$= \sigma_0\begin{pmatrix}\lambda+\frac{1}{(\omega_c\tau)^2} & -\frac{1}{\omega_c\tau}\\ \frac{1}{\omega_c\tau} & \frac{1}{(\omega_c\tau)^2}\end{pmatrix}$$

or resistivity tensor $\vec{E} = \overleftrightarrow{\rho} \cdot \vec{j}$

$$\overleftrightarrow{\rho} = \sigma^{-1} = \frac{1}{\sigma_0} \frac{1}{\left[\frac{\lambda}{(\omega_c\tau)^2} + \frac{1}{(\omega_c\tau)^2} + \frac{1}{(\omega_c\tau)^4}\right]} \begin{pmatrix} \frac{1}{(\omega_c\tau)^2} & \frac{1}{\omega_c\tau} \\ \frac{-1}{\omega_c\tau} & \lambda + \frac{1}{(\omega_c\tau)^2} \end{pmatrix}$$

$$\cong \frac{1}{\sigma_0(1+\lambda)} \begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & \lambda(\omega_c\tau)^2+1 \end{pmatrix}$$

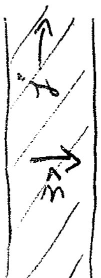
Note $\rho_{xy} = -\rho_{yx}$ as before for closed orbits, and Hall coefficient is $\frac{-\rho_{xy}}{H} = \frac{-\omega_c\tau}{\sigma_0(1+\lambda)H} = \frac{-1}{nec(1+\lambda)}$ same as before except for factor $(1+\lambda)$.

But now $\rho_{xx} \neq \rho_{yy}$. We have

expt'l wire ∥ to open orbits

ρ_{xx} — magnetoresistance for current flowing ∥ to open orbits in real space (ie $\vec{j} = j\hat{x}$)

$= \frac{1}{\sigma_0(1+\lambda)}$ ← indep of H saturates as $H \to \infty$ as in Drude model



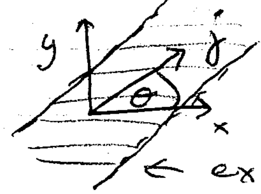
expt'l wire ⊥ to open orbits

ρ_{yy} — magnetoresistance when current flowing ⊥ to direction of open orbits in real space (ie $\vec{j} = j\hat{y}$)

$\cong \frac{\lambda}{\sigma_0(1+\lambda)}(\omega_c\tau)^2 \sim H^2$ does not saturate as $H \to \infty$. grows as H^2!

magnetoresistance which keeps increasing with H is signal for presence of open orbits on Fermi surface.

For a current in a general direction $\vec{j} = j\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$, where θ measures angle from $\hat{x}$, the direction of the open orbits in real space



← exp't'l wire at angle θ to open orbits

we have

$$\vec{E} = \overleftrightarrow{\rho}\cdot\vec{j} = \frac{j}{\sigma_0(1+\lambda)}\begin{pmatrix}\cos\theta + (\omega_c\tau)\sin\theta \\ -(\omega_c\tau)\cos\theta + (\lambda(\omega_c\tau)^2+1)\sin\theta\end{pmatrix}$$

and the longitudinal magnetoresistance is

$$\rho \equiv \frac{\vec{E}\cdot\hat{j}}{|\vec{j}|} \quad \swarrow \text{projection of } \vec{E} \text{ along current } \vec{j}.$$

$$= \frac{1}{\sigma_0(1+\lambda)}\Big[\cos^2\theta + (\omega_c\tau)\sin\theta\cos\theta - (\omega_c\tau)\cos\theta\sin\theta + [\lambda(\omega_c\tau)^2+1]\sin^2\theta\Big]$$

$$\rho = \frac{1}{\sigma_0(1+\lambda)}\Big[\underset{\uparrow}{1} + \underset{\uparrow}{\lambda(\omega_c\tau)^2\sin^2\theta}\Big]$$

constant. Drude like part from closed orbits

$\sim H^2\sin^2\theta$ increases without bound as H increases - from open orbits

Lattice Vibrations, phonons, and the speed of sound

Assume Hamiltonian of ionic degrees of freedom looks like

$$H = \underbrace{\sum_{R_i} \frac{\vec{P}_i^2}{2M}}_{\text{kinetic}} + \underbrace{\mathcal{U}_{ion}(\{\vec{R}_i\})}_{\text{potential due to ion-ion interactions}}$$

ions at positions $\vec{R}_i$, momentum $\vec{P}_i$, mass M

Write $\vec{R}_i = \vec{R}_i^0 + \vec{u}_i$

$\vec{R}_i^0$: position in periodic BL

$\vec{u}_i$: small displacement due to elastic distortions

If $\vec{u}_i$ is small, expand $\mathcal{U}_{ion}$ about the BL positions $\vec{R}_i^0$. Since the positions $\vec{R}_i^0$ are assumed to be positions of mechanical equilibrium, the linear term in the expansion must vanish, and the quadratic term is the leading order term.

$$\mathcal{U}_{ion}(\{\vec{u}_i\}) = \mathcal{U}_{ion}^0 + \frac{1}{2}\sum_{\substack{i\alpha \\ j\beta}} u_{i\alpha} \mathbb{D}_{ij}^{\alpha\beta} u_{j\beta}$$

i, j label BL sites

α, β label components x, y, z of the displacement

$$\mathbb{D}_{ij}^{\alpha\beta} = \left.\frac{\partial^2 \mathcal{U}_{ion}}{\partial u_{i\alpha}\, \partial u_{j\beta}}\right|_{\{\vec{R}_i^0\}}$$ is the <u>dynamical matrix</u>