RKKY Interaction and Spin Glasses

In our discussion of the Lindhard dielectric function we saw that:

If there is a potential energy $U(\vec{r})$ that couples to the electron density, ie the perturbation in the Hamiltonian is

$$\sum_i U(\vec{r}_i) = \int d^3r \, U(\vec{r}) \, n(\vec{r})$$

with $n(\vec{r}) \equiv \sum_i \delta(\vec{r}-\vec{r}_i)$ is the electron density then U induces a change in electron density $\delta n(\vec{r})$ given, in Fourier transform space, by

$$\delta n(\vec{q}) = \chi(\vec{q}) \, U(\vec{q})$$

with $$\chi(\vec{q}) \equiv \frac{2}{V} \sum_k \frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k} = 2 \int \frac{d^3k}{(2\pi)^3} \frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k}$$

where f_k is the Fermi occupation function for the free electron state with wave vector $\vec{k}$ and energy ε_k.

Consider a magnetic impurity with spin $\vec{S}_0$ located at position $\vec{R}_0$. We will assume the interaction of $\vec{S}_0$ with the conduction electrons is via a <u>local</u> spin-spin interaction.

$$\delta H = - J \mu_B \vec{S}_0 \cdot \underbrace{\sum_i \vec{s}_i}_{\text{spin of electron } i} \underbrace{|\psi_i(\vec{R}_0)|^2}_{\text{probability for electron } i \text{ to be at position } \vec{R}_0}$$

$$= J \vec{S}_0 \cdot \underbrace{\vec{m}(\vec{R}_0)}_{\text{magnetization density of electrons}}$$

Let us take the direction of $\vec{S}_0$ to be $\hat{z}$. Then

$$\delta H = - J \mu_B S_0 \left[\underbrace{n_\uparrow(\vec{R}_0)}_{\text{density of } \uparrow \text{ electrons}} - \underbrace{n_\downarrow(\vec{R}_0)}_{\text{density of } \downarrow \text{ electrons}} \right]$$

$$= \delta H_\uparrow + \delta H_\downarrow$$

$$\delta H_\uparrow \equiv - J \mu_B S_0 \, n_\uparrow(\vec{R}_0) = \int d^3r \; U_\uparrow(\vec{r}) \, n_\uparrow(\vec{r})$$

$$\delta H_\downarrow = + J \mu_B S_0 \, n_\downarrow(\vec{R}_0) = \int d^3r \; U_\downarrow(\vec{r}) \, n_\downarrow(\vec{r})$$

$$\text{where} \begin{cases} U_\uparrow(\vec{r}) = - J \mu_B S_0 \, \delta(\vec{r} - \vec{R}_0) \\ U_\downarrow(\vec{r}) = J \mu_B S_0 \, \delta(\vec{r} - \vec{R}_0) \end{cases}$$

We then have that $U_\uparrow$ and $U_\downarrow$ induce perturbations $\delta n_\uparrow$ and $\delta n_\downarrow$ in the spin ↑ and spin ↓ electron densities.

$$\delta n_\uparrow(\vec{q}) = \tfrac{1}{2}\chi(\vec{q})\, U_\uparrow(\vec{q})$$

$$\delta n_\downarrow(\vec{q}) = \tfrac{1}{2}\chi(\vec{q})\, U_\downarrow(\vec{q}) = -\tfrac{1}{2}\chi(\vec{q})\, U_\uparrow(\vec{q})$$

↑ factor of $\frac{1}{2}$ since $n_\uparrow$ and $n_\downarrow$ are both $\frac{1}{2}$ of total density $n = n_\uparrow + n_\downarrow$

induced electron magnetization is then

$$m_z(\vec{q}) = -\mu_B\left[\delta n_\uparrow(\vec{r}) - \delta n_\downarrow(\vec{r})\right]$$
$$= -\mu_B\, \chi(\vec{q})\, U_\uparrow(\vec{q})$$

Now $U_\uparrow(\vec{r}) = -J\mu_B S_0\, \delta(\vec{r}-\vec{R}_0)$

so $U_\uparrow(\vec{q}) = \int d^3\vec{r}\, e^{-i\vec{q}\cdot\vec{r}}\, U_\uparrow(\vec{r})$

$$= -J\mu_B S_0\, e^{-i\vec{q}\cdot\vec{R}_0}$$

$$\Rightarrow m_z(\vec{r}) = -\int \frac{d^3q}{(2\pi)^3}\, J\mu_B^2 S_0\, \chi_q\, e^{-i\vec{q}\cdot\vec{R}_0}\, e^{i\vec{q}\cdot\vec{r}}$$

$$= -J\mu_B^2 S_0 \int \frac{d^3q}{(2\pi)^3}\, \chi(\vec{q})\, e^{i\vec{q}\cdot(\vec{r}-\vec{R}_0)}$$

$$m_z(\vec{r}) = -J\mu_B^2 S_0\, \chi(\vec{r}-\vec{R}_0)$$

↑ Fourier transform of $\chi(\vec{q})$ evaluated at position $\vec{r}-\vec{R}_0$

induced magnetization is in the same direction as $\vec{S}_0$, so

$$\vec{m}(\vec{r}) = -J\mu_B^2 \vec{S}_0 \, \chi(\vec{r}-\vec{R}_0)$$

For <u>many impurities</u> $\vec{S}_i$ at positions $\vec{R}_i$, the total induced electron magnetization is obtained from the above by superposition

$$\vec{m}(\vec{r}) = -J\mu_B^2 \sum_i \vec{S}_i \, \chi(\vec{r}-\vec{R}_i)$$

The interaction Hamiltonian is then

$$\delta H = J \sum_j \vec{S}_j \cdot \vec{m}(\vec{R}_j)$$

$$\boxed{\delta H = -J^2\mu_B^2 \sum_{i,j} \vec{S}_j \cdot \vec{S}_i \, \chi(\vec{R}_j-\vec{R}_i)}$$

Above result shows how the magnetization of the conduction electrons mediates an interaction between the two magnetic impurities $\vec{S}_i$ and $\vec{S}_j$. If $\chi(\vec{R}_j-\vec{R}_i) > 0$ then the interaction is <u>ferromagnetic</u>. If $\chi(\vec{R}_j-\vec{R}_i) < 0$ then the interaction is anti<u>ferromagnetic</u>.

Now $\chi(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \chi(\vec{q})$

with $\chi(\vec{q}) = 2\int \frac{d^3k}{(2\pi)^3} \left[\frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k} \right]$

$$= g(\varepsilon_F) \left[1 + \frac{1-x^2}{2x} \ln \left| \frac{1+x}{1-x} \right| \right]$$

where $x = q/2k_F$

As discussed in connection with the Lindhard dielectric function, $\chi(\vec{q})$ has a singularity at $x=0$ or $|\vec{q}| = 2k_F$. This results in $\chi(\vec{r})$ having a piece that goes as

$$\chi(\vec{r}) \sim \frac{1}{r^3} \cos(2k_F r)$$

which oscillates in sign depending on the value of the distance r. Since the magnetic impurities $\vec{S}_i$ are randomly positioned in the metal, with an average spacing several times the atomic lattice constant, then $k_F|\vec{R}_i - \vec{R}_j|$ in general is large and hence $\chi(\vec{R}_i - \vec{R}_j)$ will be randomly positive or negative, according to the particular random separation between the spins. Thus the interaction between spins $\vec{S}_i$ and $\vec{S}_j$ is randomly ferro or anti-ferro magnetic. This is the the model interaction for a "spin glass" where the spins freeze into random orientations as T decreases