

In the preceding calculations, we treated the paramagnetic and diamagnetic effects separately. i.e., when computing Pauli paramagnetism we ignore the change in electron wavefunction due to the presence of the magnetic field H , and only considered the interaction of H with the intrinsic electron magnetic moment μ_B . When computing Landau diamagnetism we ignored this interaction with the intrinsic moment, and considered only the effect of H on the eigenstates and hence the density of states.

Of course both effects are there simultaneously, so the total magnetic susceptibility of the free electron gas is therefore

$$\chi = \chi_P + \chi_L = \chi_P - \frac{1}{3} \chi_L = \frac{2}{3} \chi_P$$

Since $\chi_P > 0$, the net effect is paramagnetic.

For some more traditional calculations of Landau diamagnetism see:

Notes from AP Young UC Santa Cruz

<http://bartok.ucsc.edu/peter/231/magnetic-field/node5.html>

Pathria - "Statistical Mechanics", pgs 206 - 209

Landau + Lifshitz - "Statistical Mechanics v1", pgs 172 - 175

In real metals and doped semiconductors one does not necessarily find that the total χ is paramagnetic. When we include effects of band structure, which causes the electron to behave as if it had an "effective" mass m^* , rather than its true free particle mass m , we can find that χ can be either paramagnetic or diamagnetic.

The Pauli paramagnetic susceptibility was

$$\chi_p \sim g(E_F) = \frac{m k_F}{k^2 \pi^2} \quad \text{for free electrons}$$

For electrons in a periodic potential, χ_p will go as $\sim m^*$ rather than m , so

$$\chi_p \sim \frac{m^*}{m} \chi_{p \text{ free electron}}$$

The Landau diamagnetic susceptibility came from the Lorentz force on the electron in a magnetic field

$$\chi_L \sim \frac{e \vec{v}}{c} \times \vec{H}$$

where velocity $v \sim \frac{dE_k}{dk} \sim \frac{1}{m}$ For electrons in a periodic potential we will then have $v \sim \frac{1}{m^*}$

$$\text{so} \quad \chi_L \sim \frac{m}{m^*} \chi_{L \text{ free electron}}$$

So $\frac{\chi_L}{\chi_p} \sim \left(\frac{m}{m^*}\right)^2 \frac{\chi_{L \text{ free}}}{\chi_{p \text{ free}}}$ Since we can have

$m^* \ll m$ in some materials, it is possible to have $|X_L| \gg X_p$ and the material will have a ~~net~~ net diamagnetic response.

The de Haas - van Alphen effect

At sufficiently low temperature and high magnetic field, so that $\hbar\omega_c > k_B T$, the oscillations due to the discrete Landau levels can be observed in measurements of magnetization $M = -\frac{1}{V} \frac{\partial E}{\partial H}$. These were first observed by de Haas and van Alphen in 1930 in magnetization measurements on Bi at 14.2°K. Similar oscillations are found in susceptibility $\chi = \frac{\partial M}{\partial H}$, conductivity (Shubnikov-de Haas effect), and many other quantities. Since we found that E_F has such oscillations, so $g(E_F)$ will have such oscillations, hence we can easily see why many physical quantities also oscillate.

The period of oscillations is in the inverse magnetic field $1/H$

$$\text{period is } \Delta X = 1 \Rightarrow \Delta \left(\frac{E_F^0}{\hbar\omega_c} \right) = 1 \quad \omega_c = \frac{eH}{mc}$$

since E_F^0 is fixed while H varies, we have oscillations that are periodic in $1/H$ with period

$$\Delta \left(\frac{1}{H} \right) = \frac{\hbar}{E_F^0} \frac{e}{mc}$$

we can rewrite this as

$$\Delta\left(\frac{1}{H}\right) = \frac{\hbar^2 m}{\hbar^2 k_F^2} \frac{e}{mc} = \frac{2e}{\hbar c k_F^2}$$

cross sectional area of the Fermi sphere
is $A_F = \pi k_F^2$, so

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c} \frac{1}{A_F}$$

The above turns out to be more generally true.

For electrons in a periodic potential (as opposed to our free electron model) the Fermi surface is not necessarily a sphere. Still the above relation holds where A_F is the ~~maximal~~ ^{extremal} cross sectional area of the Fermi surface perpendicular to the direction of the applied magnetic field. The de Haas-van Alphen effect thus became one of the methods for measuring the shape of the Fermi surface.

see Ashcroft + Mermin Chpt 14 for more details

Screening and the dielectric function of the free electron gas

If an external electrostatic potential $V(\vec{r})$ is applied to the electron gas, the average electron density will no longer be constant, as electrons will get attracted to the maximum of V (so that $-eV$ is minimized).

If we write the new spatially varying ~~charge~~ electron density as

$$n(\vec{r}) = n + \delta n(\vec{r})$$

where n is the uniform density when $V=0$, then

$$\delta \rho = -e \delta n(\vec{r}) \text{ is called the "induced"}$$

charge density.

The induced charge density $\delta \rho$ creates an "induced" electrostatic potential of its own, δV , given by,

$$-\nabla^2 \delta V = 4\pi \delta \rho \quad \text{Poisson's eqn for electrostatics}$$

So the total potential is then

$$V^{\text{tot}} = V + \delta V$$

Consider this in Fourier transform space

$V(\vec{k})$, $V^{\text{tot}}(\vec{k})$, $\delta V(\vec{k})$, $\delta \rho(\vec{k})$ are the Fourier transforms of $V(\vec{r})$, $V^{\text{tot}}(\vec{r})$, $\delta V(\vec{r})$, $\delta \rho(\vec{r})$

$$\left. \begin{aligned} V(\vec{r}) &= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} V(\vec{k}) \\ V(\vec{k}) &= \int d^3\vec{r} e^{-i\vec{k}\cdot\vec{r}} V(\vec{r}) \end{aligned} \right\} \text{etc.}$$

The dielectric function $\epsilon(\vec{k})$ is then defined by

$$V^{\text{tot}}(\vec{k}) = \frac{V(\vec{k})}{\epsilon(\vec{k})}$$

[In your EM course you probably saw the dielectric constant ϵ , defined by $D = \epsilon E$. Here we generalize this to spatially varying situations by defining $\epsilon(\vec{k})$. The limit $\epsilon(k \rightarrow 0)$ is the uniform dielectric constant you saw in EM. Recall D is the field produced by the "free" charge, i.e. the charge external to the material itself, while E includes the fields due to the polarized charges that make up the material. Hence $\vec{E} = -\vec{\nabla} V^{\text{tot}}$, while $\vec{D} = -\vec{\nabla} V$ the applied potential]

$$\text{Now } -\nabla^2 \delta V = 4\pi \delta \rho$$

$$\Rightarrow k^2 \delta V(\vec{k}) = 4\pi \delta \rho(\vec{k})$$

$$\delta V(\vec{k}) = \frac{4\pi}{k^2} \delta \rho(\vec{k})$$

$$V^{\text{tot}}(\vec{k}) = V(\vec{k}) + \delta V(\vec{k}) = V(\vec{k}) + \frac{4\pi}{k^2} \delta \rho(\vec{k})$$

$$\text{so } V(\vec{k}) = V^{\text{tot}}(\vec{k}) - \frac{4\pi}{k^2} \delta \rho(\vec{k})$$

$$\frac{V(\vec{k})}{V^{\text{tot}}(\vec{k})} = \varepsilon(\vec{k}) = 1 - \frac{4\pi}{k^2} \frac{\delta \rho(\vec{k})}{V^{\text{tot}}(\vec{k})}$$

We now need to find what is the $\delta \rho$ that is induced in the presence of a V^{tot} .

(Note: the electrons see not just the applied potential V , but also the induced potential δV . Hence, when considering the behavior of an isolated electron, it sees the total potential $V^{\text{tot}} = V + \delta V$, where δV describes the interaction of the electron under consideration with all the other electrons. This is an example of a mean field approximation — we will approximate the interaction of an electron with all other electrons via the average electrostatic potential δV created by the other electrons.)