At large \( H \) fields so that \( \omega_c \gg 1 \) we then have

\[
\vec{\tau} = \frac{1}{\sigma_0} \begin{pmatrix}
1 & \omega_c^2 \\
-\omega_c^2 & 1
\end{pmatrix}
\begin{pmatrix}
\tau_{xx} & \tau_{xy} \\
\tau_{yx} & \tau_{yy}
\end{pmatrix}
\]

\[
\tau_{yx} = -\tau_{xy}
\]

\[
\vec{E} = \vec{j} \cdot \vec{B}
\]

For \( \vec{j} = j^x \hat{x} \) then \( E_y = \tau_{yx} j^x = -\tau_{xy} j^x \)

Hall coefficient

\[
R = \frac{E_y}{j H} = -\tau_{xy} = \frac{-eB}{\sigma_0 H}
\]

\[
R = -\frac{eH}{n^* c} \frac{m^*}{e^2 c^2} \frac{1}{H} = -\frac{1}{ne c} \text{ Drude value!}
\]

So we regain the Drude prediction, but only in the limit of large \( H \), ie \( \omega_c \gg 1 \)

The above was for closed occupied orbits

If we had closed unoccupied orbits we would use instead the hole picture

Now we would have

\[
\vec{j} = +n^* e \frac{\vec{\omega}}{\omega_c^2} - n_0 e H \vec{x} \vec{\omega}
\]

where \( n^* \) is density of holes, and holes act like particles of charge \( +e \)
All results follow through just taking $e \rightarrow +e$, with perpendicular $\mathbf{H}$, $\mathbf{n} \rightarrow \mathbf{n}$, $m^* \rightarrow m^*$, and we get

$$R_{HH} = \frac{1}{n_e e C}$$

now the Hall coefficient is positive!

Magnetoresistance

$$\sigma (H) = \frac{E_x}{J} = \sigma_{xx} = \frac{1}{\sigma_0}$$

Same for electrons and holes

$$\sigma_0 = \frac{n e^2}{m^*} \ \text{electrons}, \ \sigma_0 = \frac{n_H e^2}{m_H^*} \ \text{for holes}$$

For more than one partially filled band

$$\sigma = \sigma_1 + \sigma_2 = \frac{\sigma_{10}}{w_{11} w_{21}} \left( \begin{array}{cc}
\frac{1}{w_{21}^2} & -1 \\
1 & \frac{1}{w_{11}^2}
\end{array} \right)$$

$$+ \frac{\sigma_{20}}{w_{22} w_{22}} \left( \begin{array}{cc}
\frac{1}{w_{22}^2} & -1 \\
1 & \frac{1}{w_{22}^2}
\end{array} \right)$$

For the Hall coefficient in $w_{22} \gg 1$ limit, we can ignore the diagonal terms to write

$$\sigma = \left( \frac{\sigma_{10}}{w_{11} w_{21}} + \frac{\sigma_{20}}{w_{22} w_{22}} \right) \left( \begin{array}{cc}
0 & -1 \\
1 & 0
\end{array} \right)$$

$$= \left( \frac{n_1 e C}{H} + \frac{n_2 e C}{H} \right) \left( \begin{array}{cc}
0 & -1 \\
1 & 0
\end{array} \right)$$
\[ \Omega = \frac{\text{neff} e C}{h} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]

where \( \text{neff} = n_1 + n_2 \) if both bands are electronic;
\( = n_1 - n_2 \) if band 1 is electronic,
\( \text{and } 2 \text{ is holes} \)

Hall coefficient, etc.

\[ \Rightarrow R = -\frac{1}{\text{neff} e C} \]

neff explains why \( R \) can have non-Drude values and even the opposite sign!

To get magneto-resistance we would need to keep the diagonal terms in \( \Omega_1 \) and \( \Omega_2 \). It is then messier to mutiply \( \Omega \) and get \( \Omega \). See HW1#1

For \( \text{neff} = 0 \), see text. This is the case for an undoped semiconductor.
Hall coefficient is
\[ R = -\frac{\sigma_{xy}}{\sigma_x} \] (see quantum Hall effect notes)
\[ = -\frac{\omega_e c}{\sigma_0 H} = -\frac{e}{m^*c} \frac{\tau m^*}{ne^2 \tau_c H} = -\frac{1}{m^*c} \text{ as before} \]

magneto resistance
\[ \sigma_{xx} = \sigma_{xy} = \sigma_0 \]
saturates to finite value as \( H \to 0 \)
just as was found in Drude model, except now \( m \) is dispersionless (there are several pointally filled bands).

Case (2) Neither all occupied states nor all unoccupied states have closed orbits \( \Rightarrow \) in either electron or hole picture there are open orbits we have to consider

Now we will find that the \( \langle \vec{k} \rangle \) contribution to current \( j \) from these open orbits no longer vanishes in the \( \omega_e \to \infty \) limit, and it dominates over the drift contribution to the current – new.
Note: For an open orbit traveling along \( \hat{k}_y \), \( k_y(t) \) is periodic in time \( \Rightarrow \) \( \mathcal{U}_y = \langle \frac{\partial \mathcal{E}}{\partial k_y} \rangle = 0 \) averaged over time. But \( k_x(t) \approx \) constant + oscillation \( \Rightarrow \) \( \mathcal{U}_x = \langle \frac{\partial \mathcal{E}}{\partial k_x} \rangle \neq 0 \) \( \Rightarrow \) electron moves in \( \hat{x} \) direction.]

Repeated zone scheme in \( k \)-space

\[
\begin{align*}
E_x < 0 & \Rightarrow \text{net} \\
\mathcal{U}_x > 0 & \Rightarrow j_x < 0 \\
\text{so} \quad j_x & \sim E_x \text{ to lowest order in } E \\
\frac{1}{\mathcal{J}} & \sim \hat{x} \langle \mathcal{E} \cdot \hat{x} \rangle \\
\text{[Open orbits in } \hat{k}_y \text{ direction have real space direction } + \hat{x} \Rightarrow \text{they gain energy from field if } E_x < 0 \text{ as energy absorbed is } - \epsilon \mathcal{E} \cdot \hat{x} \text{ between collisions.}} \\
\text{Open orbits in } -\hat{k}_y \text{ directions have real space direction } -\hat{x} \Rightarrow \text{they lose energy if } E_x > 0. \\
\end{align*}
\]

We assume therefore that the imbalance in occupation of open orbits in steady state gives rise to a current. If \( \hat{m} \) is the direction in real space of the open orbits, then the contribution to current \( \mathcal{J} \) is in the \( \hat{m} \) direction, and proportional to some function of \( \mathcal{E} \cdot \hat{m} \).

Therefore:

\[
\frac{1}{\mathcal{J}} \sim \hat{m} g(\mathcal{E} \cdot \hat{m}) \quad \text{expand in small } \mathcal{E}. 
\]
Equivalently, since \( \bar{E} = E - \hbar k \cdot \vec{v} \) is conserved between collisions, if \( \Delta E = -e \bar{E} \cdot \Delta \vec{k} \) is energy absorbed by electron from \( E \)-field then
\[
\Delta \bar{E} = 0 \Rightarrow \Delta E = \hbar \vec{w} \cdot \Delta \vec{k}
\]

So again we see in our example that it is the right hand open orbits moving along \( \hat{k}y \) that absorb energy, i.e. \( \vec{w} \cdot \Delta \vec{k} > 0 \) for these orbits, while \( \vec{w} \cdot \Delta \vec{k} < 0 \) for left hand open orbits moving along \( -\hat{k}y \).

So both \( \vec{w} \cdot \Delta \vec{k} \) and \( -E \cdot \vec{v} \) tell how much energy the electron absorbs from \( E \)-field.
This imbalance in steady state occupation of open orbits is determined by the quantity \(-e\mathbf{E} \cdot \mathbf{v}_c\), the energy absorbed by electron from \(\mathbf{E}\)-field in between collisions. If \(\mathbf{A}\) is real space direction of open orbit, \(\Rightarrow \mathbf{v} \parallel \mathbf{A}\) in \(\mathbf{A}\) direction, so the current due to open orbits is in the \(\mathbf{A}\) direction, and is some function of \((\mathbf{E} \cdot \mathbf{A})\).

\[
\mathbf{j}_{\text{open}} = \mathbf{A} g(\mathbf{E} \cdot \mathbf{A})
\]

- \(g(\mathbf{E} \cdot \mathbf{A})\) is expanded for small \(\mathbf{E}\), using
  - \(g(0) = 0\) when \(\mathbf{E} = 0\), and
  - \(g(\mathbf{E}) = -g(-\mathbf{E})\)

\[
\mathbf{j}_{\text{open}} \sim \mathbf{A} (\mathbf{A} \cdot \mathbf{E})
\]  
where proportionality constant is independent of magnetic field \(\mathbf{H}\)

We can write the contribution to conductivity tensor due to open orbits as

\[
\mathbf{j}_{\text{open}} = \mathbf{\Sigma} \cdot \mathbf{E}
\]

where \(\mathbf{\Sigma} = \lambda \sigma_0 \mathbf{A} \mathbf{A}
\)

constant independent of \(\mathbf{H}\)

If we choose \(\mathbf{A} \parallel \mathbf{x}\) direction

\[
\mathbf{\Sigma} = \lambda \sigma_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}
\]

If we treat the contribution to conductivity tensor from closed orbits as before, we get for total conductivity tensor

\[
\mathbf{\Sigma} = \frac{\sigma_0}{(\omega_c t)^2} \begin{pmatrix} 1 - \omega_c^2 & \omega_c^2 t \\ \omega_c^2 t & 1 \end{pmatrix} + \lambda \sigma_0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}
\]

\[
= \sigma_0 \begin{pmatrix} \lambda \left(\frac{1}{(\omega_c t)^2} \right)^2 & -\frac{1}{\omega_c t} \\ \frac{1}{\omega_c t} & \frac{1}{(\omega_c t)^2} \end{pmatrix}
\]
or resistivity tensor $\mathbf{\bar{E}} = \mathbf{\bar{\sigma}} \cdot \mathbf{\bar{B}}$

$$\mathbf{\bar{\sigma}} = \sigma^{-1} = \frac{1}{\sigma_0} \left[ \frac{1}{(\omega_c t)^2 + (\omega_c e)^2 + (\omega_c t + \omega_c e)^2} \right] \left( \begin{array}{cc} \frac{1}{\omega_c t} & -\frac{1}{\omega_c e} \\ \frac{1}{\omega_c e} & \frac{1}{\omega_c t + \omega_c e} \end{array} \right)$$

$$\approx \frac{1}{\sigma_0 (1 + \lambda)} \left( \begin{array}{cc} 1 & \frac{\omega_c e}{\lambda(\omega_c t)^2 + 1} \\ -\frac{\omega_c e}{\lambda(\omega_c t)^2 + 1} & \frac{1}{\omega_c t + \omega_c e} \end{array} \right)$$

Note $\sigma_{xy} = -\sigma_{yx}$ as before for closed orbits, and Hall coefficient is $\frac{\sigma_{xy}}{\sigma_0 (1 + \lambda)} = -\frac{1}{n e c (1 + \lambda)}$ same as before except for factor $(1 + \lambda)$.

But now $\sigma_{xx} \neq \sigma_{yy}$. We have

$\sigma_{xx}$ - magneto-resistance for current flowing $\parallel$ to open orbits in real space (i.e. $\mathbf{j} = \mathbf{\hat{x}} \mathbf{\hat{x}}$)

$$\exp{\frac{\chi_{xx}}{\sigma_0 (1 + \lambda)}} \text{ saturates as } H \to \infty \text{ as in Drude mode}$$

$\sigma_{yy}$ - magneto-resistance when current flowing $\perp$ to direction of open orbits in real space (i.e. $\mathbf{j} = \mathbf{\hat{y}} \mathbf{\hat{y}}$)

$$\exp{\frac{\chi_{yy}}{\sigma_0 (1 + \lambda)}} = \frac{\lambda}{\sigma_0 (1 + \lambda)} (\omega_c t)^2 \sim H^2 \text{ does not saturate as } H \to \infty \text{, grows as } H^2!$$

magneto-resistance which keeps increasing with $H$ is signal for presence of open orbits on Fermi surface.
For a current in a general direction \( \vec{J} = j(\cos \theta) \hat{\xi} \), where \( \theta \) measures angle from \( \hat{\xi} \), the direction of the open orbits in real space, we have

\[
\vec{E} = \vec{F} \times \vec{J} = j \left( \frac{\cos \theta + (\omega e_2) \sin \theta}{\sigma_0 (1 + \lambda)} \right) \left( \begin{array}{c} \cos \theta + (\omega e_2) \sin \theta \\ -(\omega e_2) \cos \theta + (\lambda (\omega e_2)^2 + 1) \sin \theta \end{array} \right)
\]

and the longitudinal magnetoresistance is

\[
\sigma \equiv \frac{\vec{E} \cdot \vec{J}}{|J|} = \frac{1}{\sigma_0 (1 + \lambda)} \left[ \cos^2 \theta + (\omega e_2) \sin \theta \cos \theta \\ -(\omega e_2) \cos \theta \sin \theta + [\lambda (\omega e_2)^2 + 1] \sin \theta \right]
\]

\[
\sigma = \frac{1}{\sigma_0 (1 + \lambda)} \left[ 1 + \lambda (\omega e_2)^2 \sin^2 \theta \right]
\]

*Constant.*

Oude like part from closed orbits

\( \nu H^2 \sin^2 \theta \)

increases without bound as \( H \)

increases - from open orbits.
Lattice vibrations, phonons, and the speed of sound

Assume Hamiltonian of conic degrees of freedom looks like

\[ H = \sum_{\mathbf{R}_i} \frac{\mathbf{p}^2_i}{2M} + U_{\text{ion}}(\mathbf{r} \mathbf{R}_i \mathbf{R}_i) \]

Lennard-Jones potential due to ion-ion interactions

ions at positions \( \mathbf{R}_i \), momentum \( \mathbf{p}_i \), mass \( M \)

Write \( \mathbf{R}_i = \mathbf{R}_i^0 + \mathbf{u}_i \)

\( \uparrow \) position in periodic BL

small displacement due to elastic distortions

If \( \mathbf{u}_i \) is small, expand \( U_{\text{ion}} \) about the BL positions \( \mathbf{R}_i^0 \). Since the positions \( \mathbf{R}_i^0 \) are assumed to be positions of mechanical equilibrium, the linear term in the expansion must vanish, and the quadratic term is the leading order term.

\[ U_{\text{ion}}(\sum \mathbf{u}_i \mathbf{R}_i) = U_{\text{ion}}^0 + \frac{1}{2} \sum_{i \neq j} U_{ij} \mathbf{D}_{ij} \mathbf{u}_i \mathbf{u}_j \]

\( i, j \) label BL sites

\( \alpha, \beta \) label components \( x, y, z \) of the displacement

\[ \mathbf{D}_{ij}^{\alpha\beta} = \frac{\partial^2 U_{\text{ion}}}{\partial u_i^{\alpha} \partial u_j^{\beta}} \]

the dynamical matrix