PHY 103: Standing Waves in an Air Column

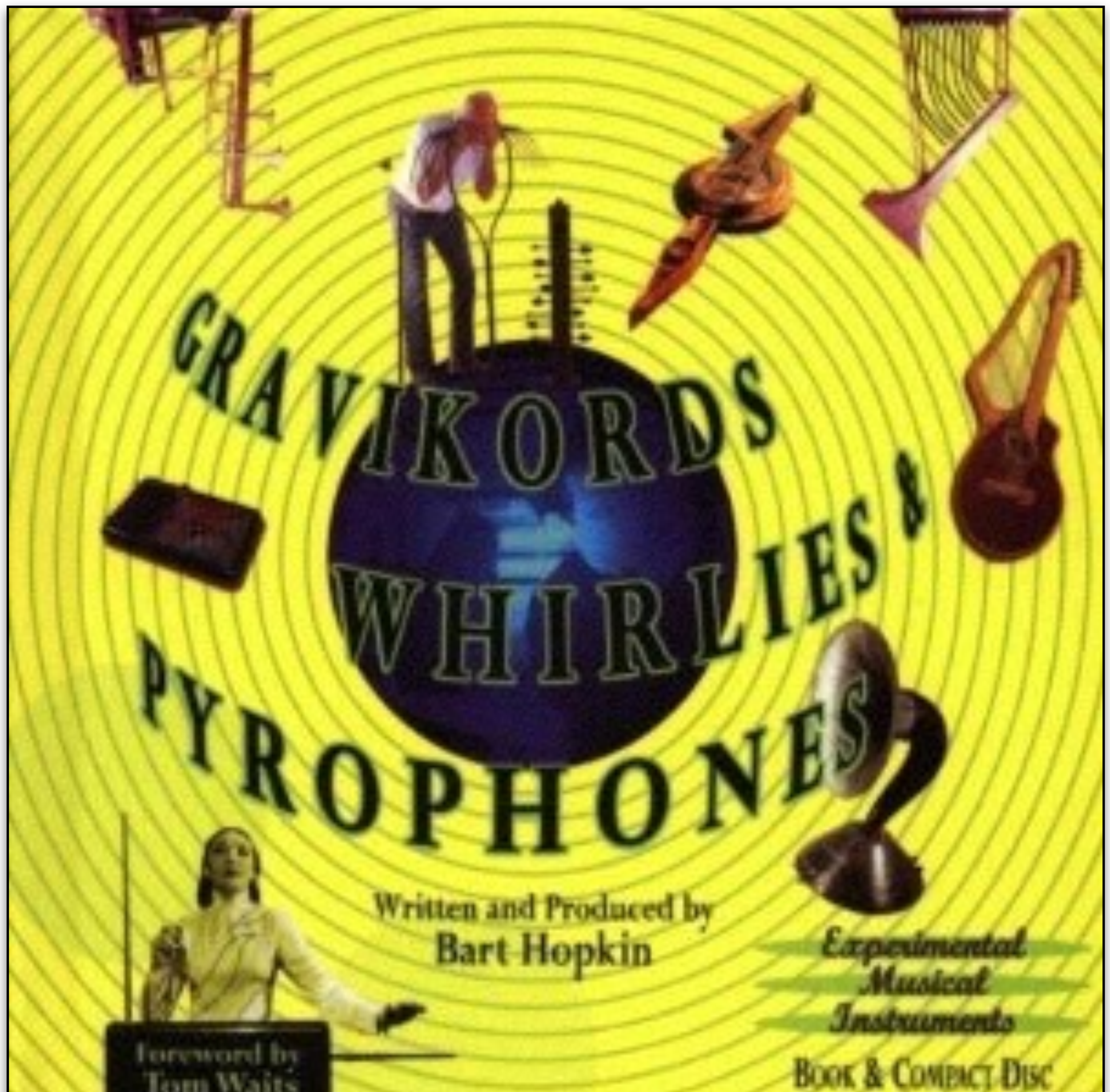
Segev BenZvi
Department of Physics and Astronomy
University of Rochester

Last Time…

- The partials present in a complex tone contribute to the timbre of the sound. Partials can be *harmonic* (ideal integer multiples of the fundamental frequency) or *inharmonic*
- High-frequency components affect the brightness of a sound
- Fourier's Theorem: any reasonably continuous periodic function can be expressed in terms of a sum of sinusoidal functions
- Spectrograms plot the square modulus of the Fourier coefficients vs. time (power spectrum)
- Sampling and the Nyquist Limit: a waveform sampled at rate f_s can be reconstructed up to frequency $f_s/2$

Song of the Day

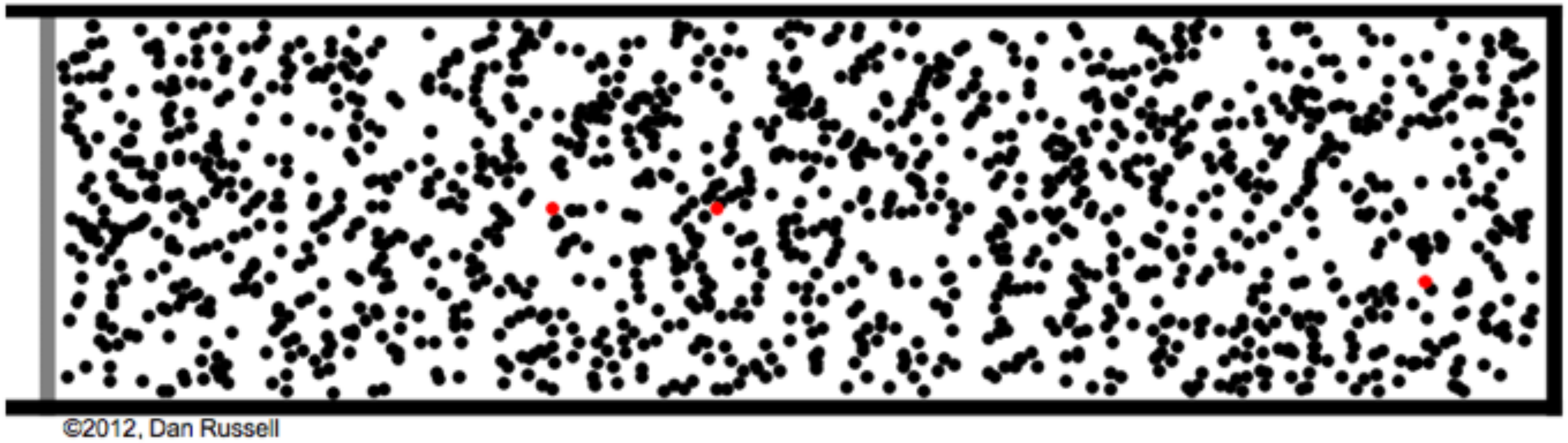
- Produced using a "whirlie" (google Sarah Hopkins)

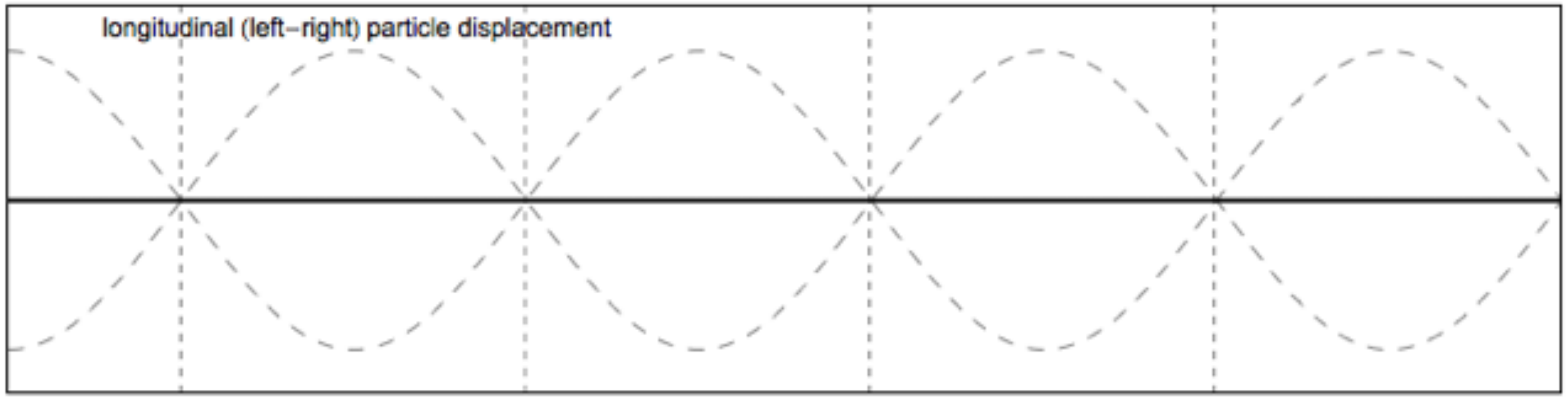


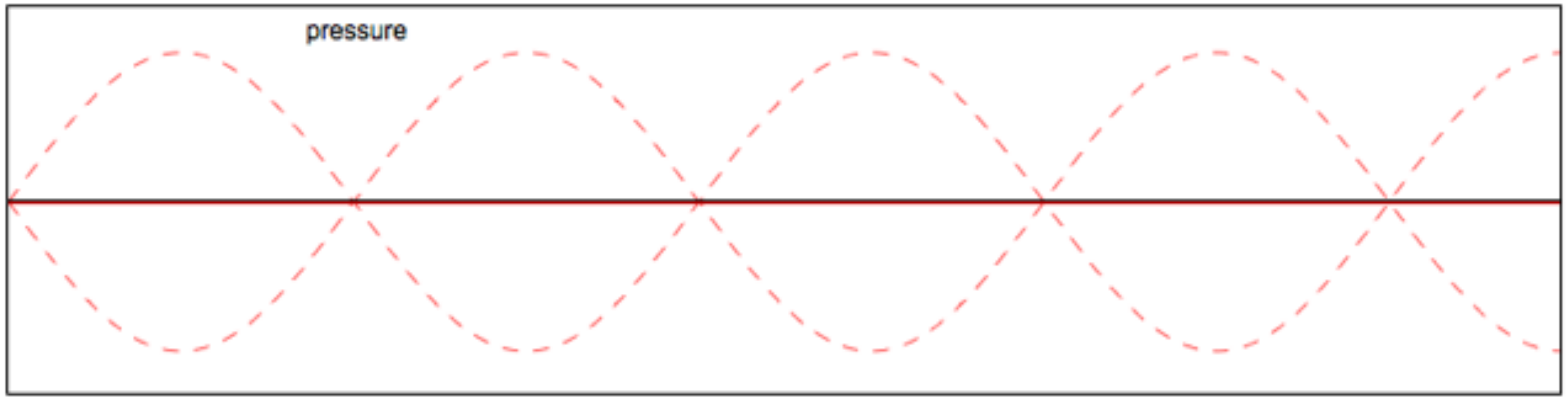
Waves in an Air Column

- Today we'll talk about standing waves in a column of air
- This is quite analogous to the topic of standing waves on a string, which we covered in detail during the past two classes
- However, your intuition has to change a bit:
  - The string supported transverse waves
  - An air column supports longitudinal waves

Waves in an Air Column

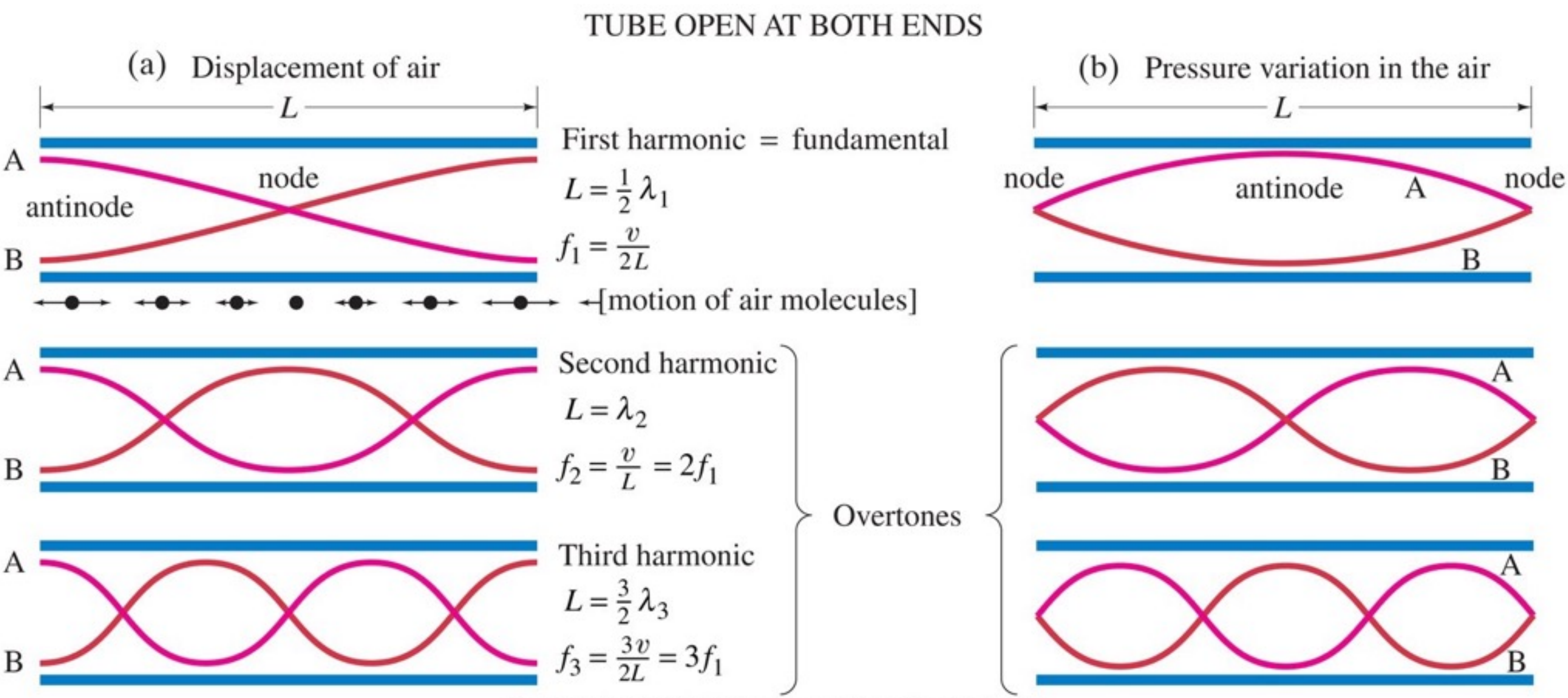






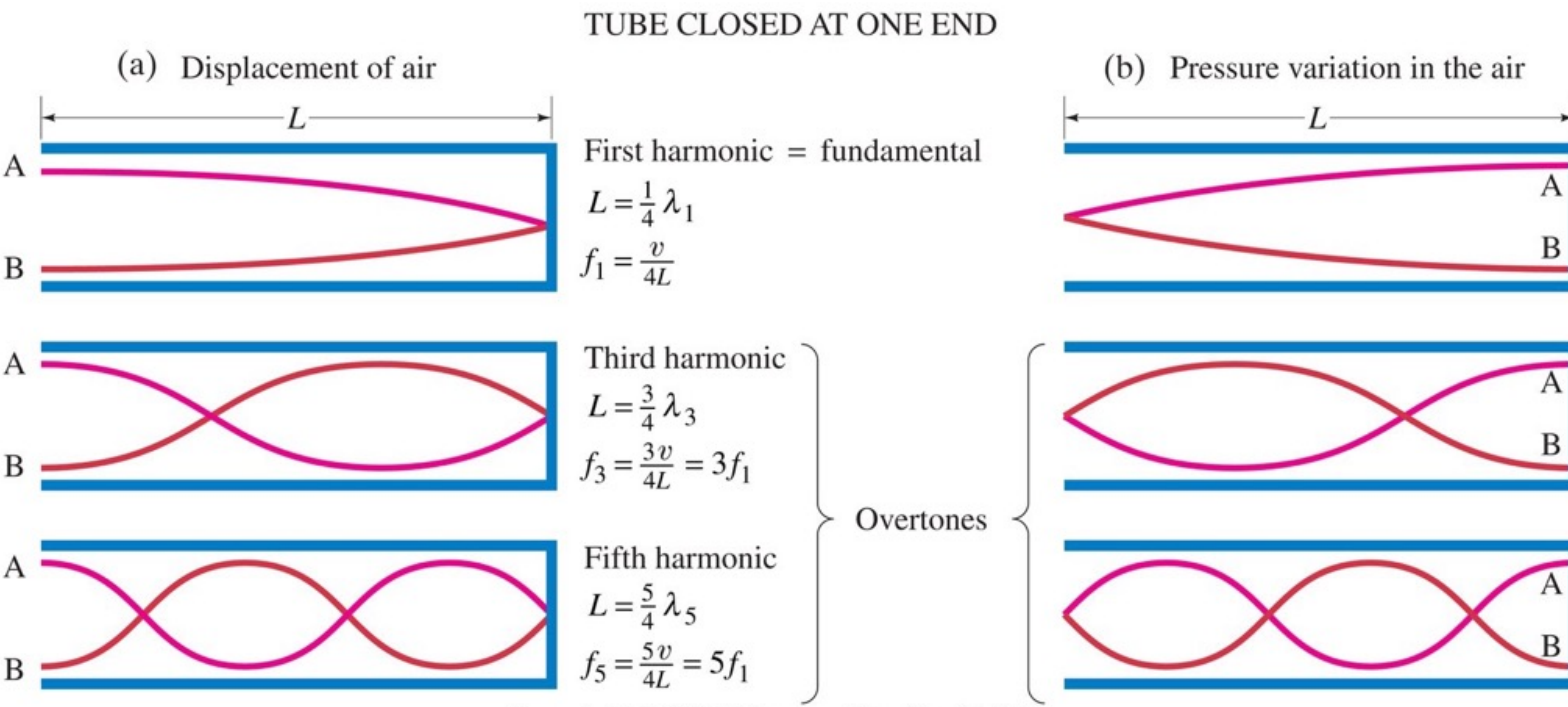
Displacement and Pressure

- Open tube (e.g., a flute) supports these waves:



Displacement and Pressure

- A closed tube (e.g., a clarinet) supports these waves:

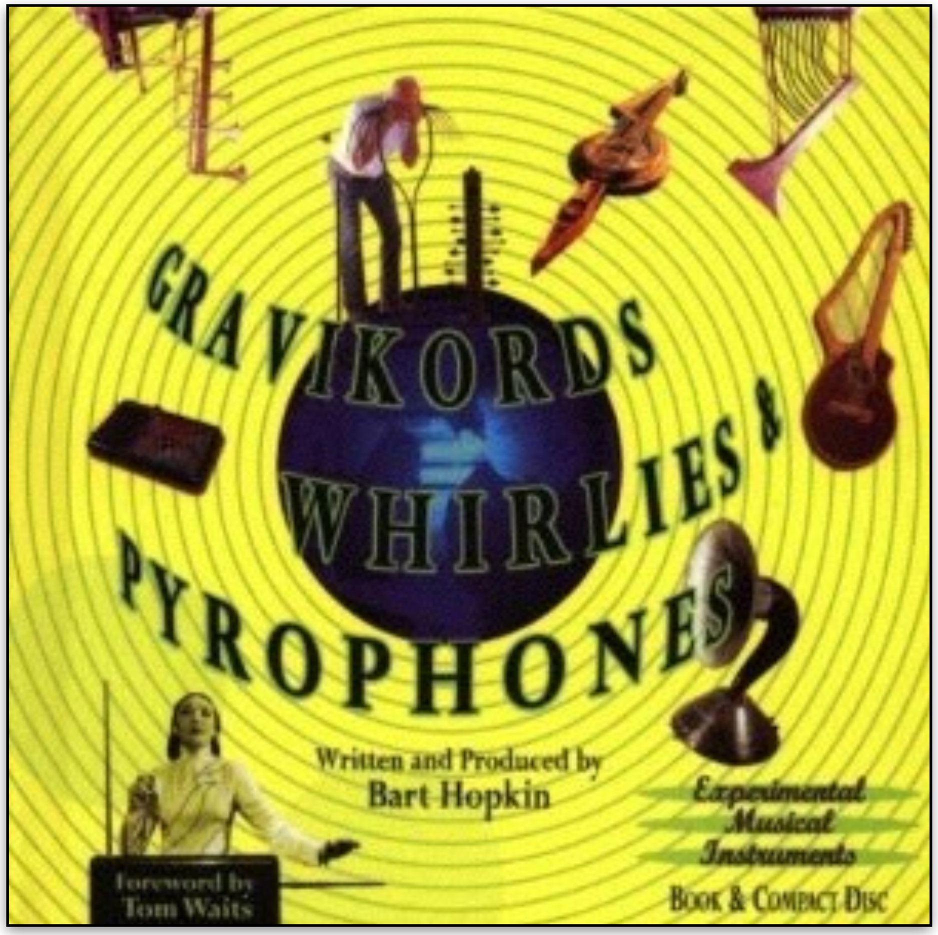


Harmonics/Overtones

- The closed tube (clarine) only supports odd harmonics: f, $3f$, $5f$, …
- The open tube (flute, pipe organ) supports all integer harmonics: f, $2f$, $3f$, $4f$, $5f$, …
- The tubes may be the same but the boundary conditions vary
  - Closed end: allows high pressure but no motion
  - Open end: allows high motion but no changes in pressure to match exterior pressure

Music from Pipe Overtones

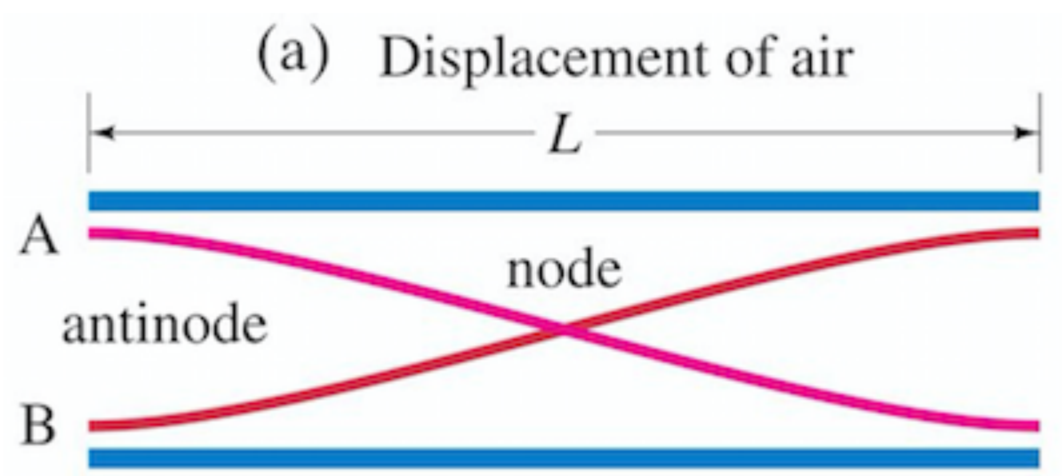
- The "whirlies" in this album are played by exciting the overtones of one or two pipes simultaneously



- Spin the pipe faster and you raise the pitch

Fundamental Tones

- Which has the lower fundamental tone? An tube open on both ends or a tube with one closed end?

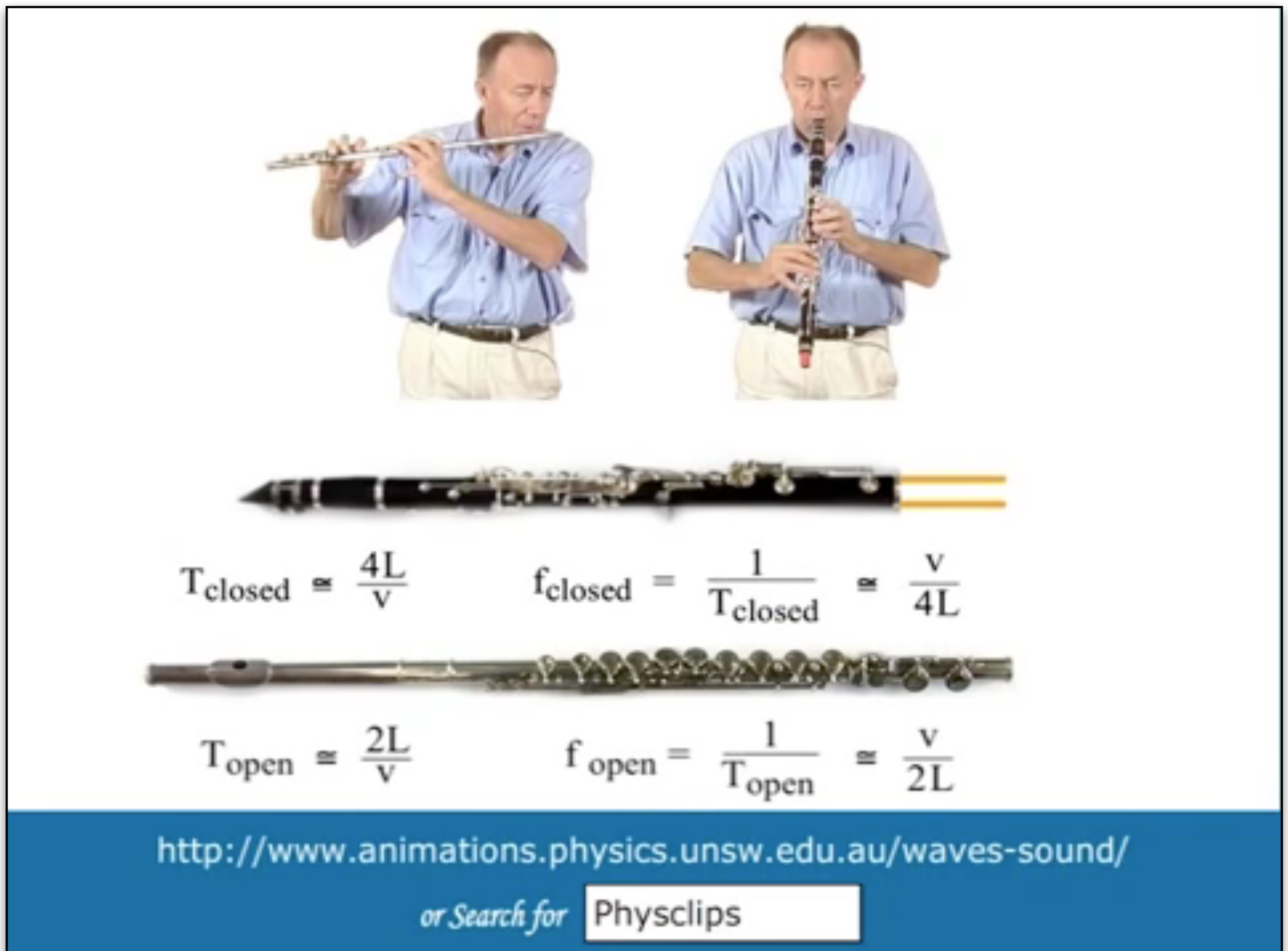


- Really nice demo of clarinet and flute with switched mouthpieces at UNSW
- http://newt.phys.unsw.edu.au/jw/flutes.v.clarinets.html

Clarinet vs. Flute

Clarinet vs. Flute

- Note the difference in the fundamental tones and the harmonics supported



Open/Closed Ends

- Effective a tube closed one one end:
  - Reed instruments
  - Horns
  - Didgeridoo
  - Panflute (depends on construction)

- Effectively a tube of air open on both ends:
  - Flutes
  - Organ pipes
  - Recorders and whistles

Speed of Sound in Air

- Recall the speed of waves on a string:

$$v = \sqrt{\frac{F_T}{\rho}}$$

- We can guesstimate an equivalent for air
  - Energy of air molecules: $E \sim k_B T$
  - Kinetic energy goes like $E \sim mv^2$
  - Equate the energy and solve for v:

$$v = \sqrt{\frac{k_B T}{m}}$$

Speed of Sound in Air

- Plug in some numbers:

$$M_{1\ \text{mol}} = \rho_{\text{air}} \cdot V_{1\ \text{mol}} = 1.2754\ \text{kg/m}^{-3} \cdot 22.4\ \text{L}$$
$$= 0.02857\ \text{kg}$$

$$m = \frac{M}{N_A} = \frac{0.02857\ \text{kg}}{6.022 \cdot 10^{23}} = 4.74 \cdot 10^{-26}\ \text{kg}$$

$$v = \sqrt{\frac{k_B T}{m}} = \sqrt{\frac{1.38 \cdot 10^{-23}\ \text{J/K} \times 293\ \text{K}}{4.74 \cdot 10^{-26}}} \approx 300\ \text{m/s}$$

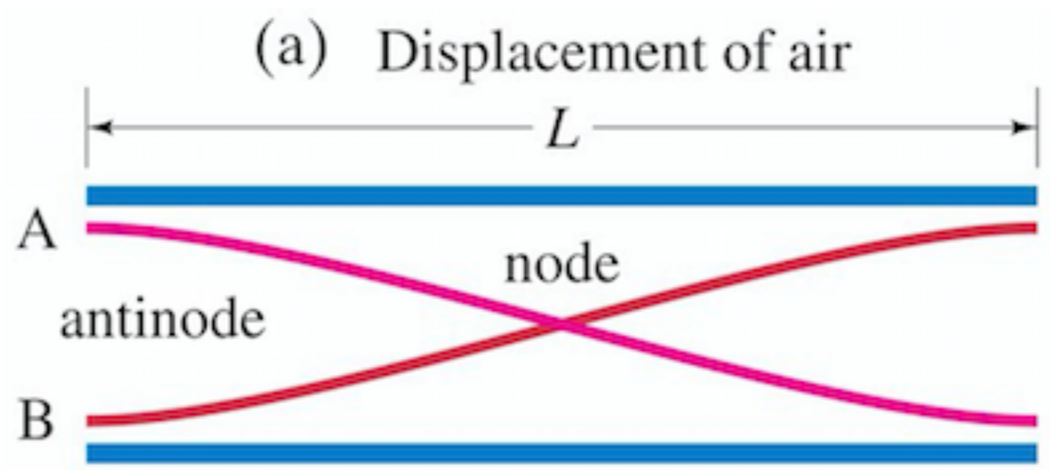
- Not bad! The real number is given by the Newton-Laplace formula:

$$v = \sqrt{\gamma \cdot \frac{k_B T}{m}},\ \text{where}\ \gamma = 1.4 \Rightarrow v \approx 343\ \text{m/s}$$

Frequency of Open Pipe

- The fundamental frequency of an open-open pipe is

$$f = v / 2L$$



- At 20 C, v=343 m/s, so for a 1 meter open pipe

$$f \approx (343 \text{ m/s}) / (2 \cdot 1 \text{ m})$$
$$= 171.5 \text{ Hz}$$

Frequency of Closed Pipe

- The fundamental frequency of an closed pipe is

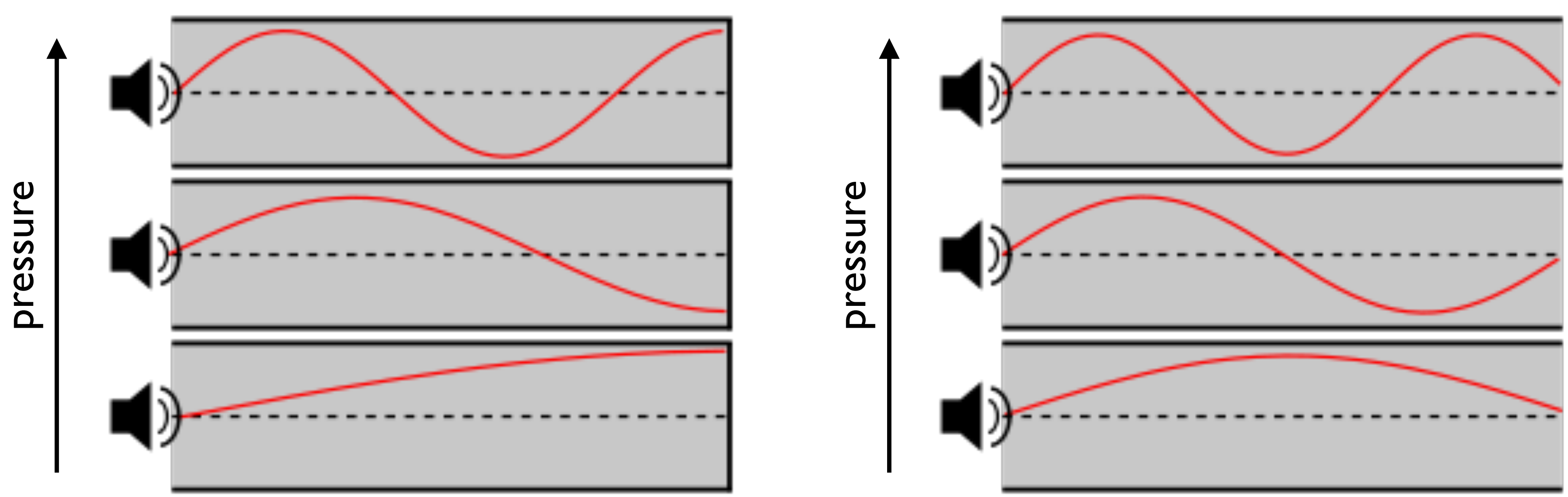
$$f = v / 4L$$

- At 20 C, *v*=343 m/s, so for a 1 meter closed pipe

$$f \approx (343 \text{ m/s}) / (4 \cdot 1 \text{ m})$$
$$= 85.75 \text{ Hz}$$

Acoustic Resonance

- If a system is driven at one of its natural vibrational frequencies, or normal modes, it will amplify the input
- Example: using a speaker to drive air in a pipe



- Correctly timed excitations cause the mode to grow

Reflections at Endpoints

- Waves reflecting off open or closed boundaries:

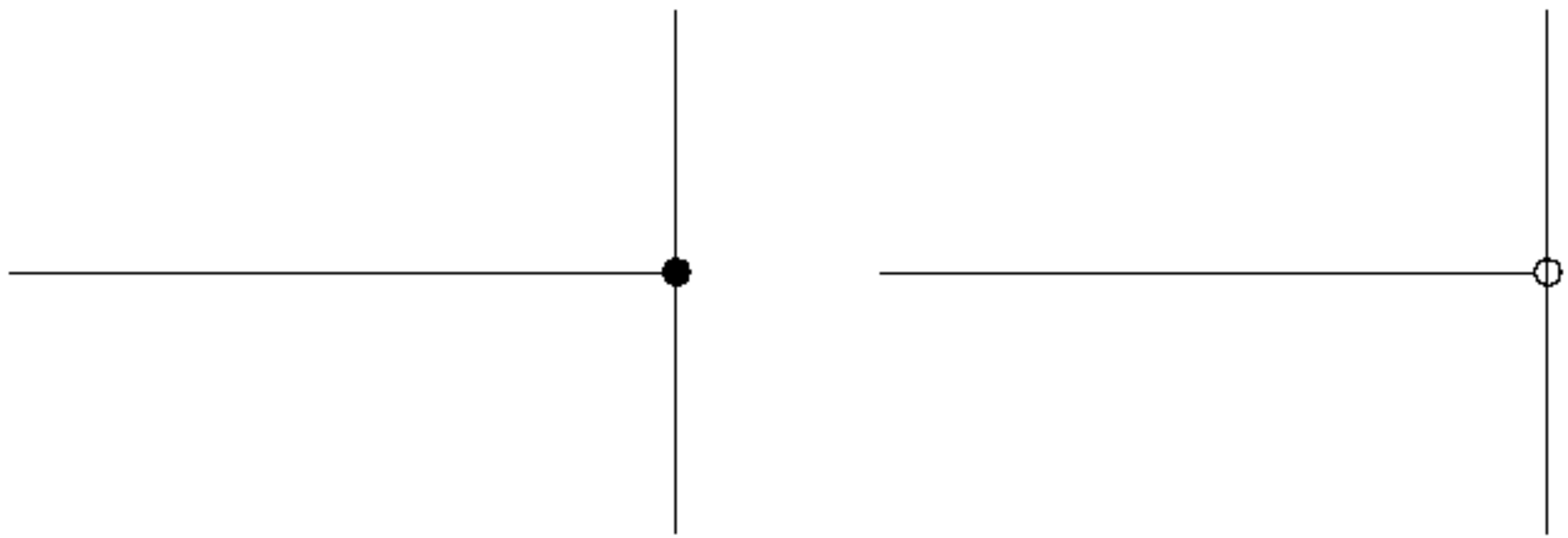


Image courtesy of Dan Russell, Graduate Program in Acoustics, PSU

- At a closed boundary, the wave will be phase shifted by 180 degrees

Reflections at Boundaries

- Sign of the wave depends on the nature of the boundary
- If the sign is the opposite or the same at both boundaries then two reflections are needed to recover the original wave
- If the sign is opposite on one side and the same on the other then four reflections are needed

End Correction for Pipe

- Reflections of sound waves on a real pipe do not occur exactly at the physical boundary, but slightly beyond it
- In other words, the pipe's acoustic length is slightly longer than its physical length by an amount **Δ***L*=0.8*d* (open pipe), where *d* is the diameter of the pipe
- As a result, the frequencies of the closed and open pipes are

$$f_{\text{open}} = \frac{nv}{2(L+0.8d)} \qquad f_{\text{closed}} = \frac{nv}{4(L+0.4d)}$$

Summary

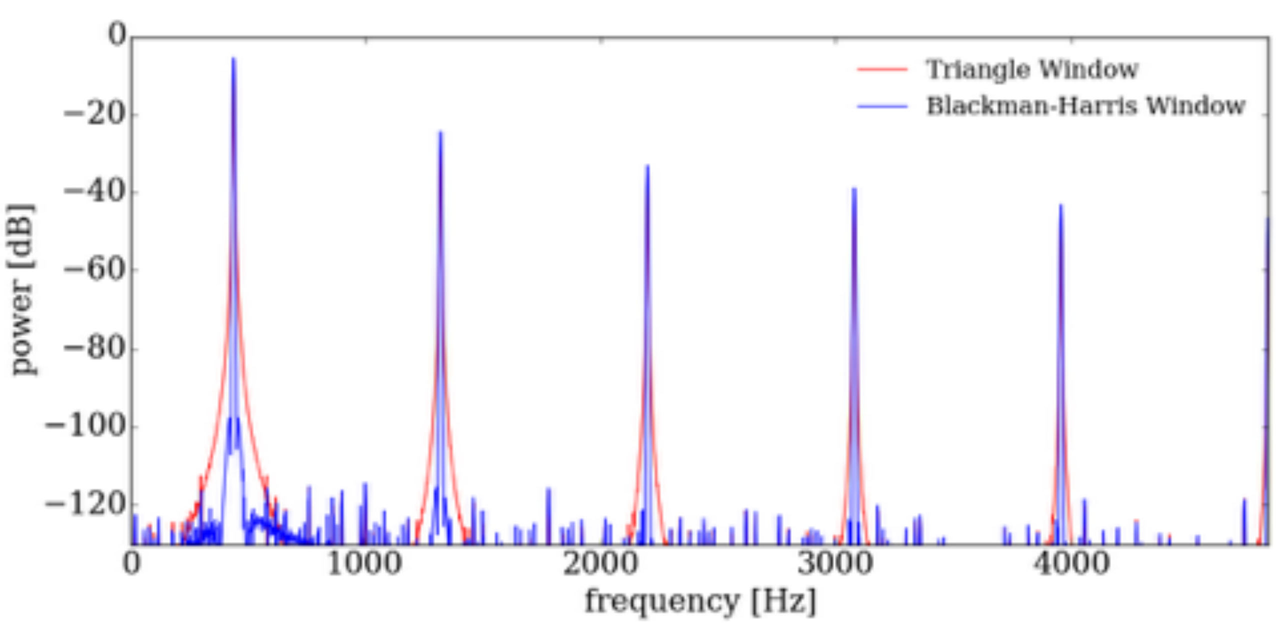
- Normal modes of open/closed pipes:
  - Half-wavelength fundamental in open pipes
  - Quarter-wavelength fundamental in closed pipes
  - Longitudinal pressure and displacement waves
- Resonances: particular input frequencies where a pipe (or any acoustic system) results in amplification
- Boundary conditions: reflections which affect acoustic behavior such as resonances
  - Correspondence bet. physical and acoustic lengths may not be exact, requiring edge corrections

Fast Fourier Transform (FFT)

- The Adobe Audition program (and it's freeware version Audicity) will perform a Fourier decomposition for you
- On the computer we can't represent continuous functions; everything is discrete
- The Fourier decomposition is accomplished using an algorithm called the Fast Fourier Transform (FFT)
  - Works really well if you have N data points, where N is some power of 2: $N = 2^k$, $k = 0, 1, 2, 3, \ldots$
  - If N is not a power of two, the algorithm will pad the end of the data set with zeros
- Details: Numerical Recipes in C, W. Press et al., 1992

Calculating the FFT

- When you calculate an FFT, you have freedom to play with a couple of parameters:
  - The number of points in your data sample, *N*
  - The window function used



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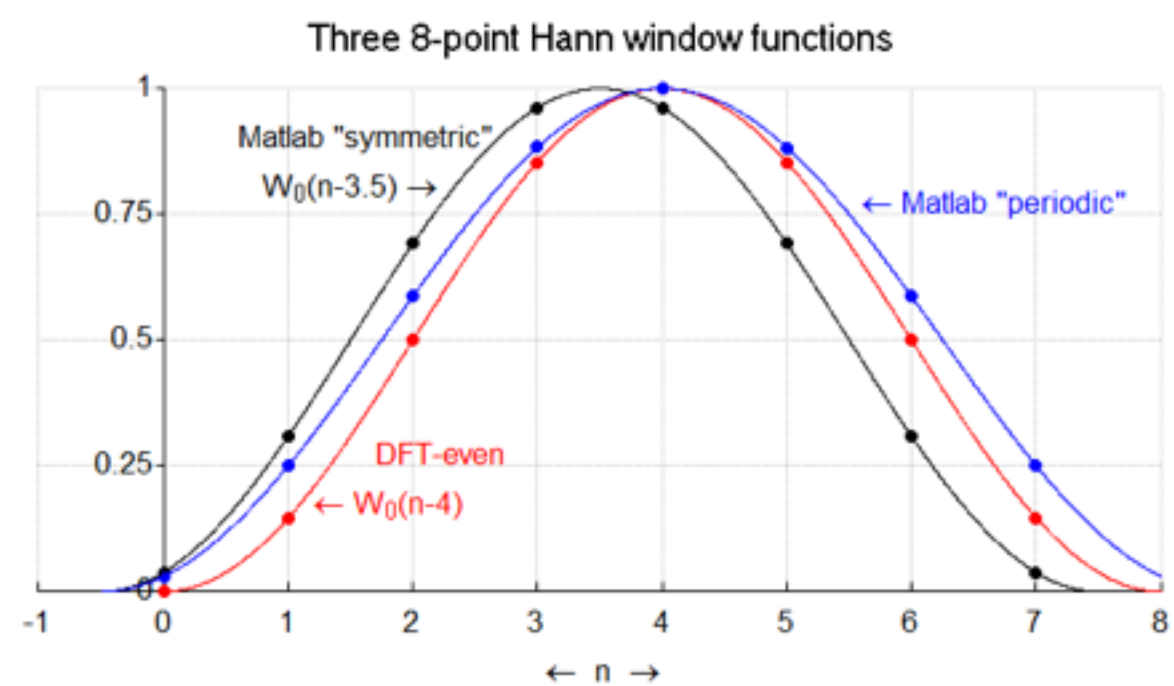
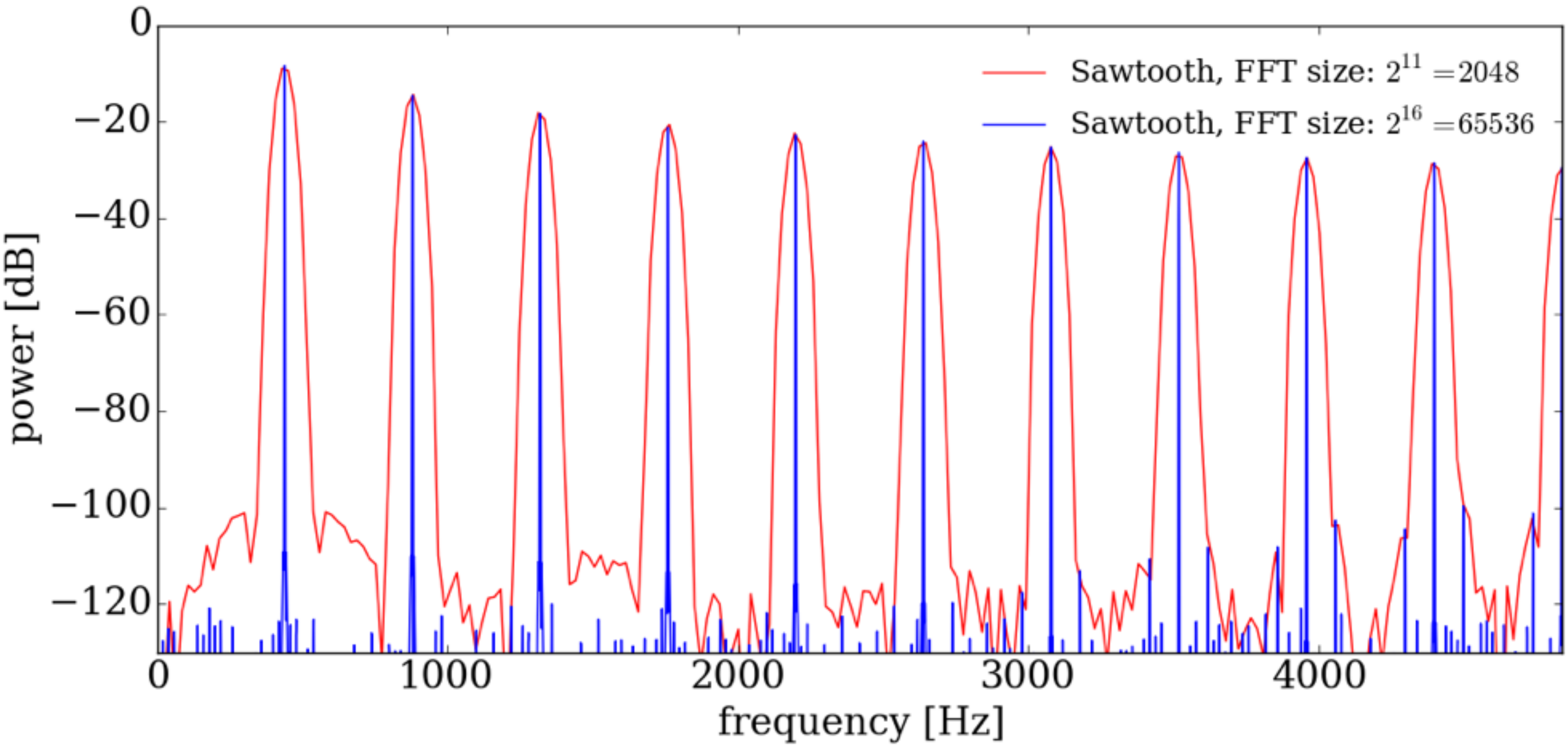


Effect of FFT Size

- Larger *N* = better resolution of harmonic peaks



Uncertainty Principle

- Why does a longer data set produce a better resolution in the frequency domain?
- Time-Frequency Uncertainty Principle:

$$\Delta t \cdot \Delta f \sim 1$$

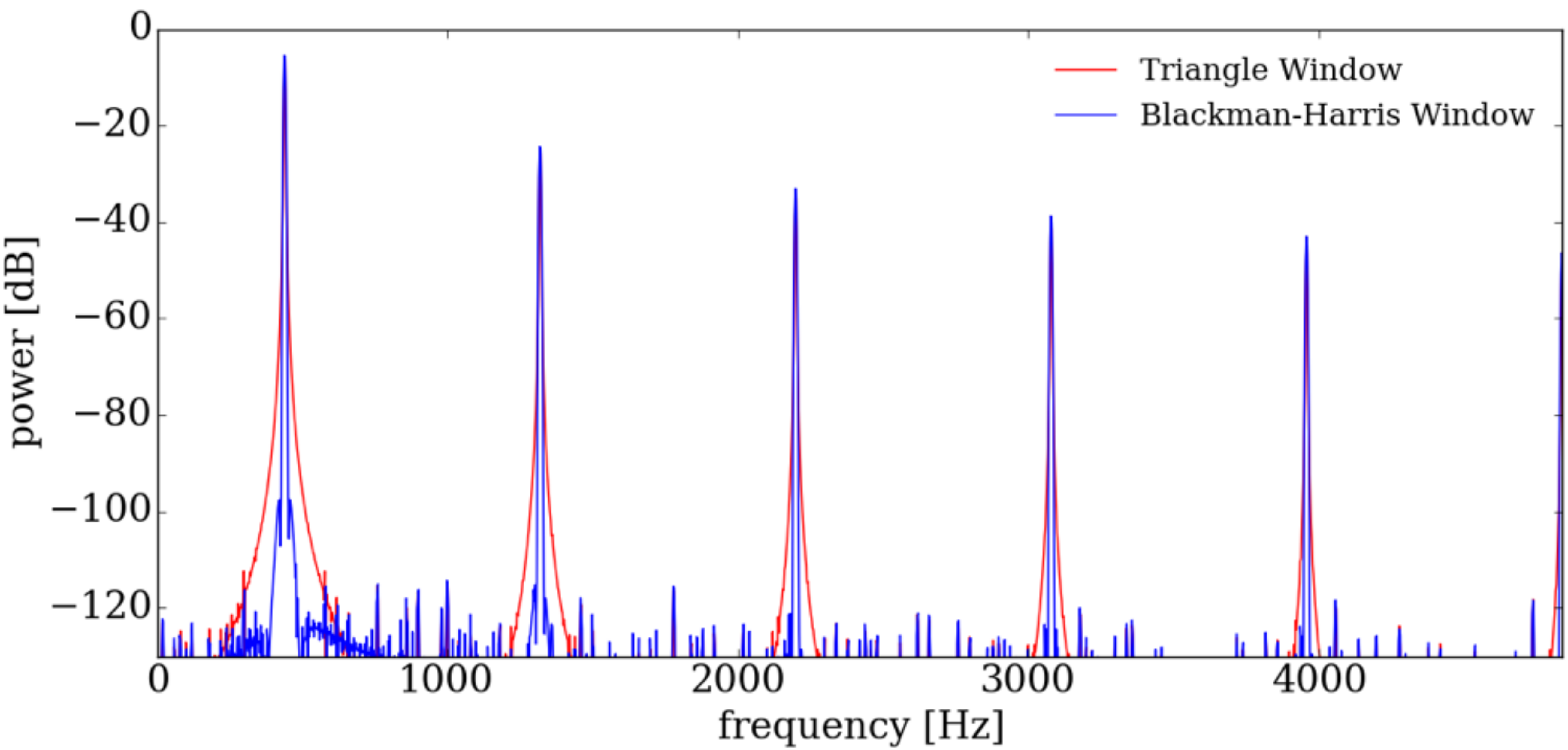
Localization of measurement in *time*

Localization of measurement in *frequency*

- Localizing the waveform in time (small N, and therefore small $\mathbf{\Delta}t$) leads to a big uncertainty in frequency ($\mathbf{\Delta}f$)
- Localizing the frequency (small $\mathbf{\Delta}f$) leads means less localization of the waveform in time (large $\mathbf{\Delta}t$)

Effect of Window Function

- Certain windows can give you better frequency resolution



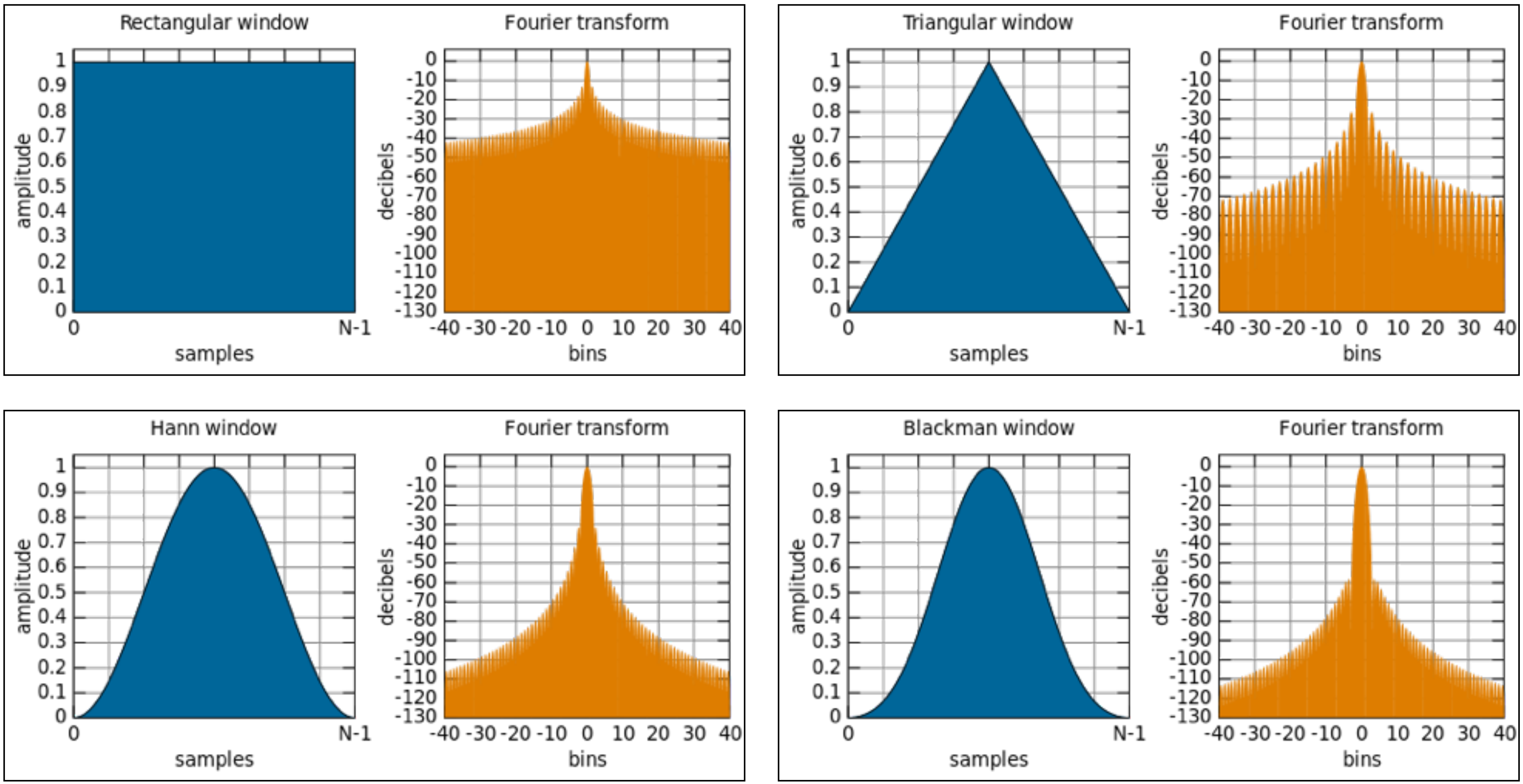
Windowing

- Why do we use a window function at all?
  - Because the Fourier Transform is technically defined for periodic functions, which are defined out to $t = \pm\infty$
  - We don't have infinitely long time samples, but truncated versions of periodic functions
  - As a result, the FFT contains artifacts (sidebands) because we've "chopped off" the ends of the function at $\pm\infty$
  - The window function mitigates the sidebands by going smoothly to zero in the time domain
  - Thus, our function doesn't drop sharply to zero at the start and end of the sample, giving a "nicer" FFT

Window Examples

- Time and frequency behavior of common windows:

Olli Niemitalo, commons.mediawiki.org



Fipple Flute

- Air from the source oscillates back and forth across the knife edge by the motion of the standing wave inside the air column