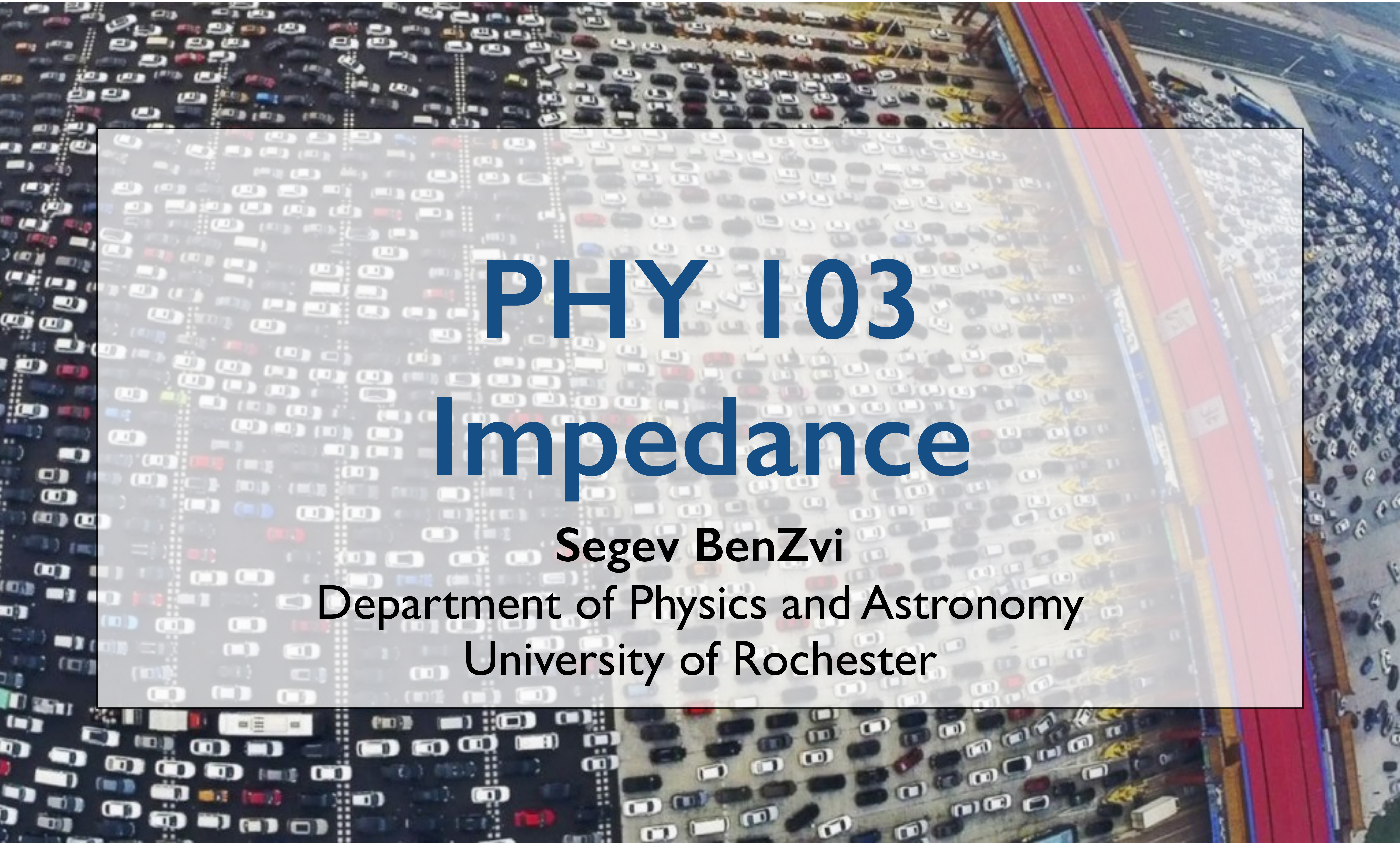

PHY 103
Impedance

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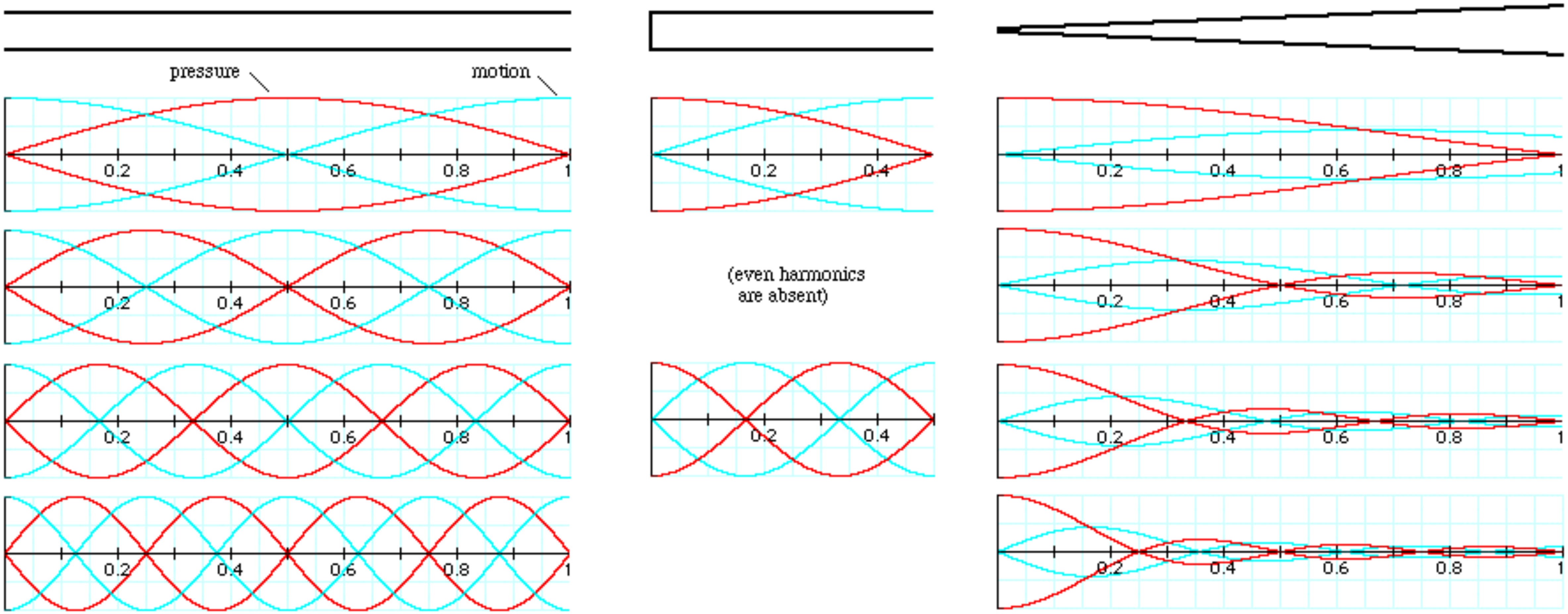
Reading

- Reading for this week:
  - Hopkin, Chapter 1
  - Heller, Chapter 1

Waves in an Air Column

- Recall the standing waves supported in air columns:



J.Wolfe, UNSW FAQ

- Note the effect of the shape of the bore
- Volume varies with position along the length of the bore

Acoustic Impedance

- If I put random fluctuations of pressure into a pipe, some modes will grow and some modes won't
- We need a way to describe how a pipe/column of air reacts to an input sound
- Impedance: relation of input pressure *p* to air volume flow rate ("velocity") *U*

$$Z = p / U$$

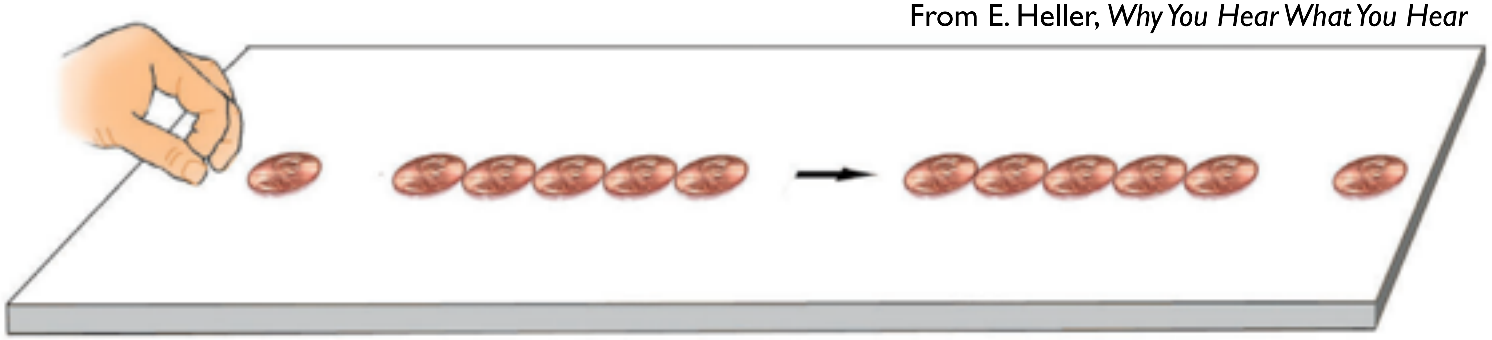
- Tells us how much a given instrument will resist the flow of air at a particular frequency *f*

Impedance in General

- Impedance is a concept used in many different areas of physics and engineering:
  - Acoustics
  - Electrical circuits
  - Hydrology
  - Mechanics
- Basically, impedance tells us how easy (or hard) it is to transfer energy between one body and another.
- Matching impedances makes it easy to transfer energy. This is extremely important for playing instruments

Impedance Matching

- We can demonstrate impedance matching with a simple desktop test of elastic collisions

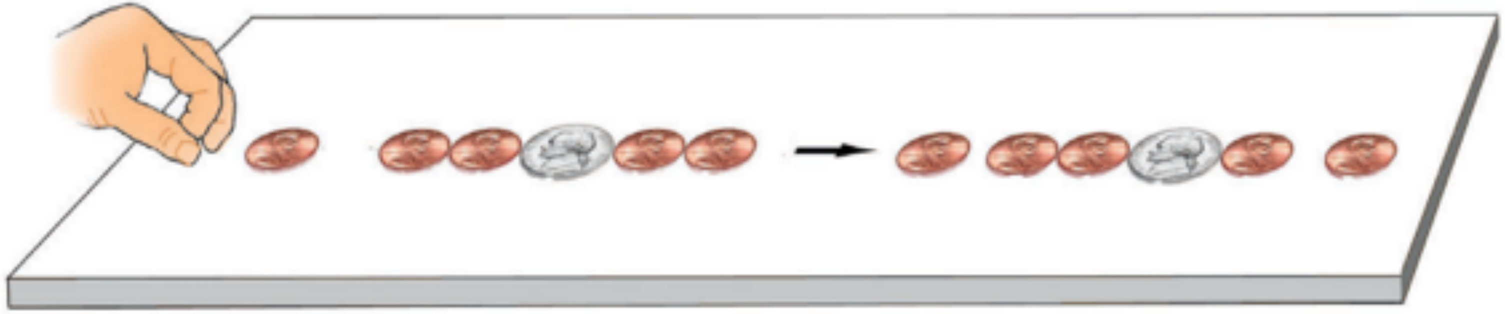


From E. Heller, *Why You Hear What You Hear*

- Collide one penny with another. The first penny will stop completely and transfer all its momentum to the second penny
- The same thing happens with a line of pennies
- The pennies are impedance matched; kinetic energy is transferred very efficiently in this elastic collision

Impedance Mismatch

- Now suppose we replace one penny with a nickel. What will happen when they collide?



From E. Heller, *Why You Hear What You Hear*

- Energy and momentum are conserved, but the velocity of the nickel is less than it would be if it were a penny
- Moreover, the first penny bounces back slightly (reflects) after the collision
- There is an impedance mismatch; not all of the energy is transferred from one body to the other

Elastic Collisions

- Recall from High School physics…
- Conservation of momentum

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2$$

- Conservation of energy

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v'^2_1 + m_2 v'^2_2$$

- Velocities after collision (penny/nickel target @ rest)

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

- Note: if the masses are the same,

$$v'_1 = 0 \qquad v'_2 = v_1$$

Reflection/Transmission

- If the two masses are not the same (e.g., 1=penny and 2=nickel) then the energy of the penny after the collision is nonzero:

$$\frac{1}{2}m_1 v'^2_1 = \frac{1}{2}\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 v_1^2$$

- The fraction of energy retained by mass 1 after the collision is

$$E'_1 / E_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

- If $m_2 = 10m_1$, this fraction is ~67%, which means 33% of the initial energy of the penny is transmitted to the nickel, and 67% is reflected backward, since $v'_1 = -9/11\ v_1$

Mechanical Impedance

- Mechanically, impedance *Z* measures the response of a body to a force *F* according to

$$Z = F / v$$

- Apply fixed force *F* to move an object from rest
  - Heavy objects have lower *v*: high impedance
  - Light objects have higher *v*: low impedance
- Matched impedance implies equal and opposite forces, so velocity lost = velocity gained, as in the case of two elastically colliding pennies

Mitigating Reflections

- Suppose we have two mismatched masses m_1 and m_2
- Kinetic energy will not be transferred completely in a collision due to the impedance mismatch
- But, we can put a third coin between them with mass $m_1 < m^* < m_2$ to mitigate the impedance mismatch
- It turns out that $m^* = \sqrt{(m_1 \cdot m_2)}$, the geometric mean of the two masses, provides the optimal energy transfer between coins 1 and 2
- The presence of m^* is an anti-reflection strategy

Application to Music

- This kind of impedance matching is used all the time in instrument design
- Example: in a violin, the impedance of the body is much higher than the impedance of the string
  - String is "coin 1"
  - Body is "coin 2"
  - The bridge plays the role of the "intermediate coin" that optimally matches the string and body

Reflection, Revisited

- Abrupt changes in impedance at a boundary lead to low energy transmission across the boundary
- Change of impedance from Z_1 to Z_2 causes fraction of energy R to be reflected

$$R = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2} \right)^2$$

- Transmitted energy is

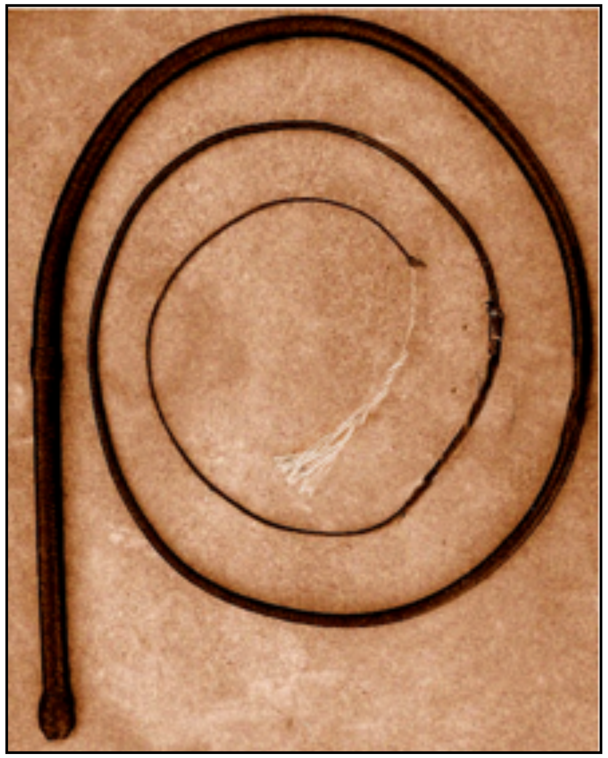
$$T = 4 \frac{Z_1 Z_2}{(Z_1 + Z_2)^2}$$

- Total energy is conserved: $R + T = 1$

Well-Matched Impedance

- A bullwhip is gradually tapered and is designed to have no abrupt changes in density or stiffness
- Handle energy $E=\frac{1}{2}MV^2$ is transferred to popper, which has energy $E'=\frac{1}{2}mv^2$
- Velocity ratio: $v/V=\sqrt{(M/m)}=20$
- If v=40 mph, V=800 mph! Supersonic!

From E. Heller, *Why You Hear What You Hear*



Undesired Matching

- Undesired impedance match: Wolf notes in cello/violin
  - Normally, $Z_{bridge} \sim 10 \times Z_{string}$, but if the body has a resonance near a low frequency mode of the string, it can severely vibrate the bridge and effectively lower Z_{bridge}
  - Result: bow and body dump vibrational energy into each other, creating a howling beat (wolf note)

Measuring Impedance

- A good way to measure impedance is to drive a system (e.g., mass + spring) with a sinusoidal force

$$F(t) = F\sin(2\pi ft)$$

- At a given frequency *f*, the ratio of the maximum force applied *F* to the maximum speed achieved *v(f)* is the impedance

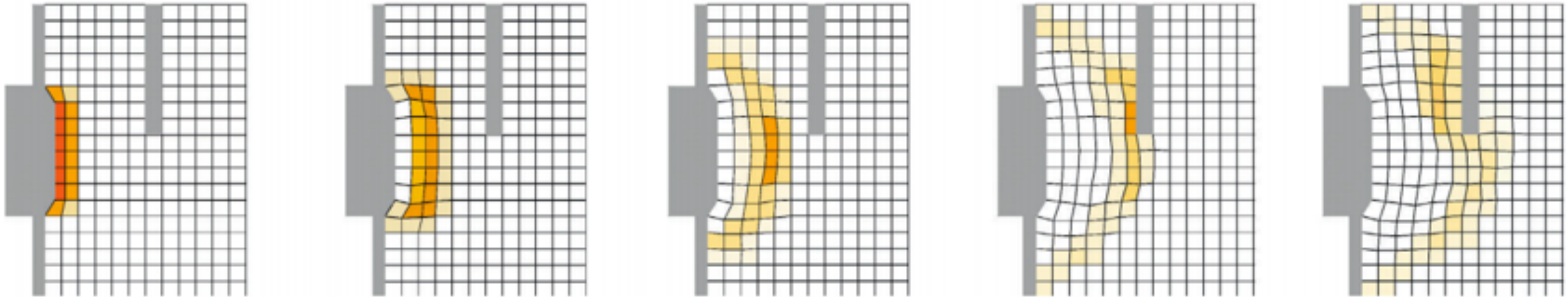
$$Z(f) = \frac{F}{v(f)}$$

- High speed *v(f)*: low impedance
- Low speed *v(f)*: high impedance

Note: you will measure the impedance of a pipe in the lab using this technique

Impedance of Air

- Imagine "driving" air with a sinusoidal force, produced by a vibrating speaker or a piston



From E. Heller, *Why You Hear What You Hear*

- Think of the volume of air as divided into little cells; each has a mass and elasticity/springiness associated with it
- A wave propagates as the air in each cell pushes on its neighbors
- If a cell pushes back too hard (higher impedance than its neighbor), it pushing neighbor recoils, producing a reflection

Acoustical Impedance

- In fluids like air, the unit of force is pressure *p*
- The flow *u* is the speed with which a cell moves due to the pressure *p*
- The specific acoustical impedance of a cell, by analogy with mechanical impedance, is

$$z = p / u$$

- The impedance of a larger collection of cells, like in a column of are, can be lumped together into a single acoustical impedance

$$Z = p / U$$

Acoustical Impedance

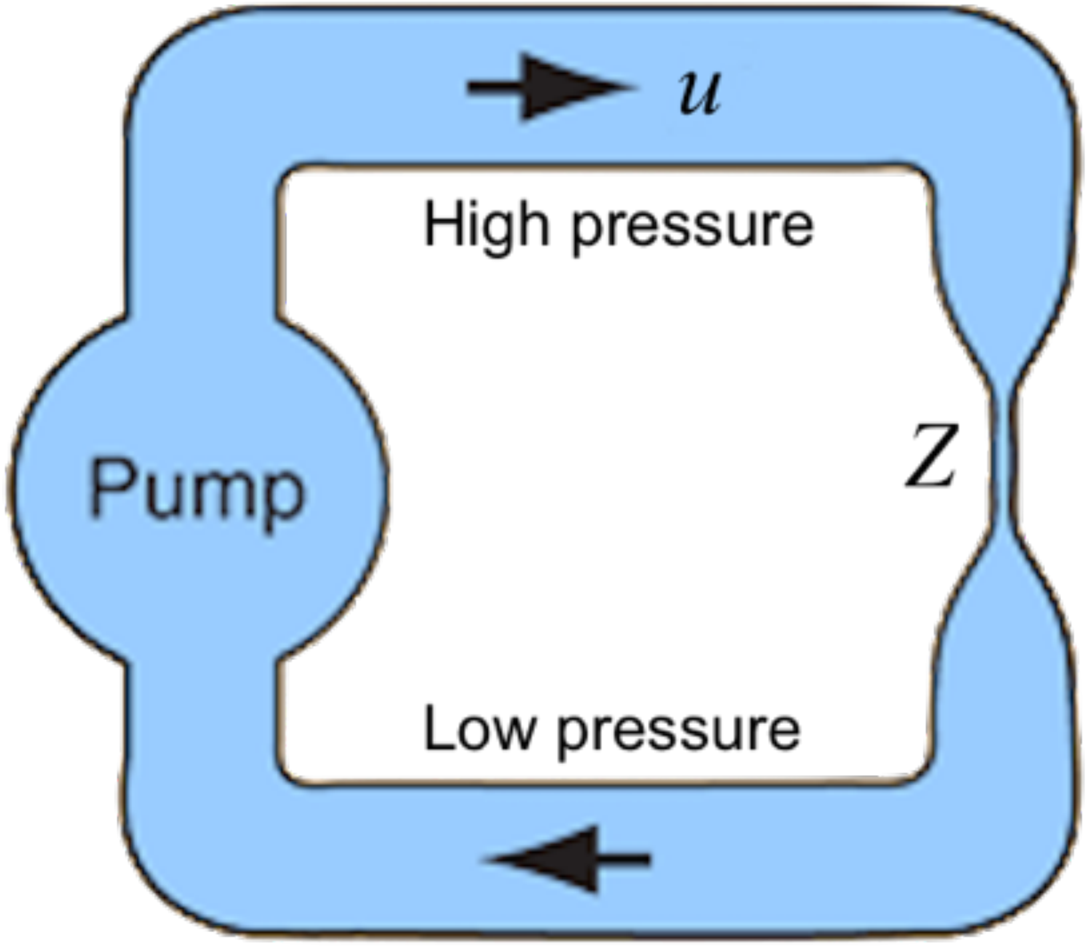
- Imagine a pipe of cross-sectional area *A*
- If the wavelength of the disturbance is much bigger than the diameter of the pipe, then we can define the uniform volume velocity $U = u \times A$
- The acoustic impedance of the pipe is

$$Z = \frac{p}{U} = \frac{p}{cA} = \frac{\rho_0 c}{A}$$

- Note: c = speed of sound, ρ = density of air
- The impedance of a pipe is inversely proportional to the area of the pipe

Hydrological Analogy

- Imagine a closed water "circuit" in which a pump moves water through a loop of pipe. Flow rate *U* is constant in the loop

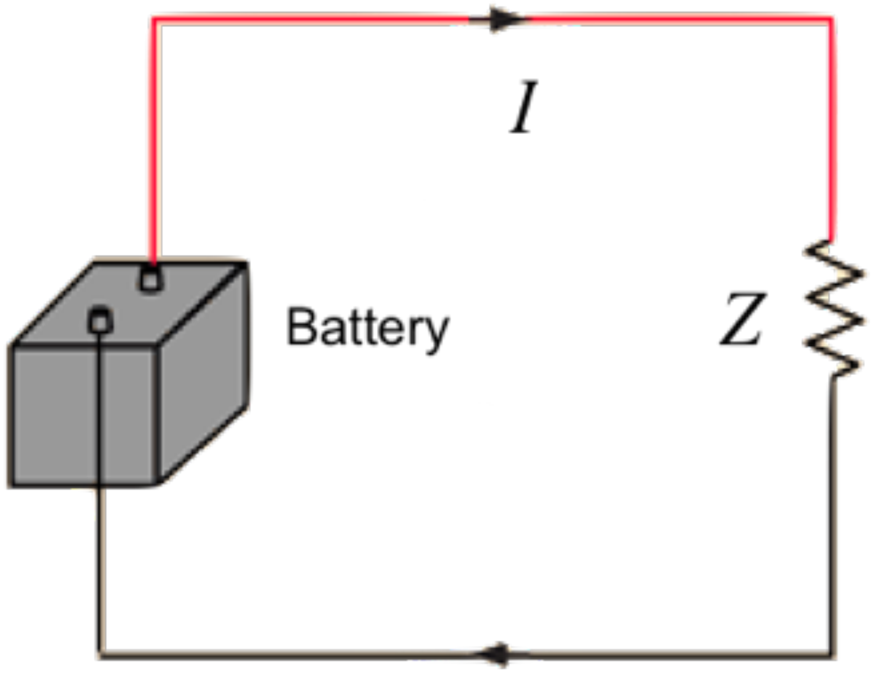


HyperPhysics

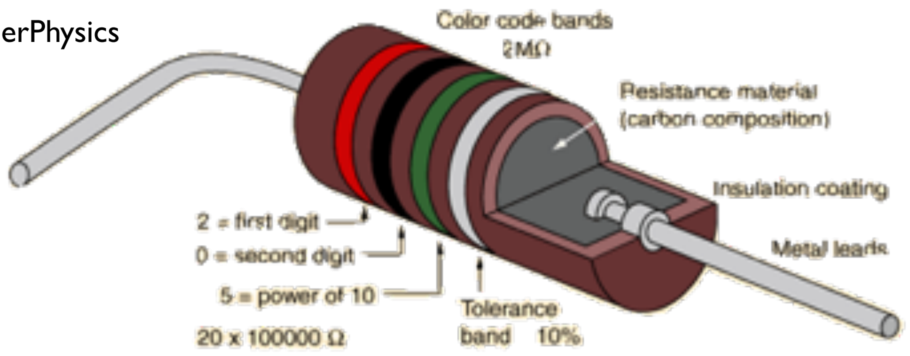
- And, there can't be any net pressure change in the closed loop
- The pump raises the pressure by Δp
- The pressure drops in the "neck" by $-\Delta p = UZ$

Electrical Analogy

- Replace the pump with a battery and the pipe with a wire. Imagine the "neck" is now an electrical resistor. The current *I* is constant in the loop



HyperPhysics

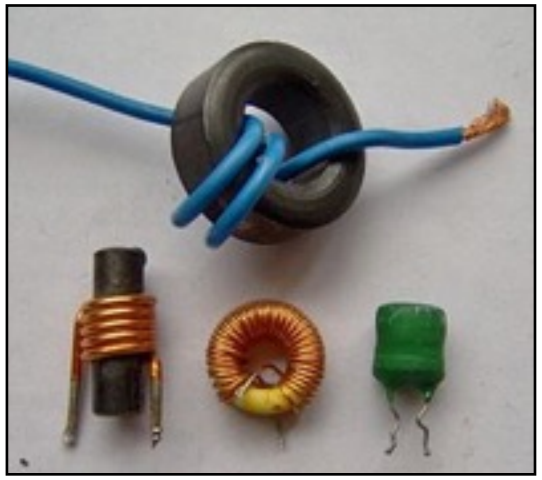


- And, the net voltage around the closed loop is 0
- The voltage between the battery terminals is ΔV
- The voltage drops across the resistor by $-\Delta V = IZ$ (Ohm's Law)

Frequency Dependence

- Frequency dependent impedance in electronics:
- Capacitor: stores charge

  - Blocks current when it's full
  - Capacitors like rapidly varying signals (AC) and block DC
- Inductor: coil wound around a magnet



  - Resists changes in electric current
  - Lets through DC signals, but blocks AC

Voltage/Pressure Analogy

- Pump: take a volume of water or air in a circuit at low pressure, do work on it, eject it at high pressure
- Battery: take charges (electrons) at low voltage, do work on them, eject them at high voltage
- Pressure: energy/volume
- Voltage: energy/charge
- Neck: *impedes* water/air flow, drops pressure
- Resistor: *impedes* current flow, drops voltage

Ohm's Law

- Ohm's Law for electrical circuits:
  - $\Delta V = IZ$
- Ohm's Law for hydrological circuits (applies to acoustics):
  - $\Delta p = UZ$
- Impedance is a generic concept describing an element that impedes the flow of current, resulting in a change of pressure (or voltage) in a circuit

Analogies Summarized

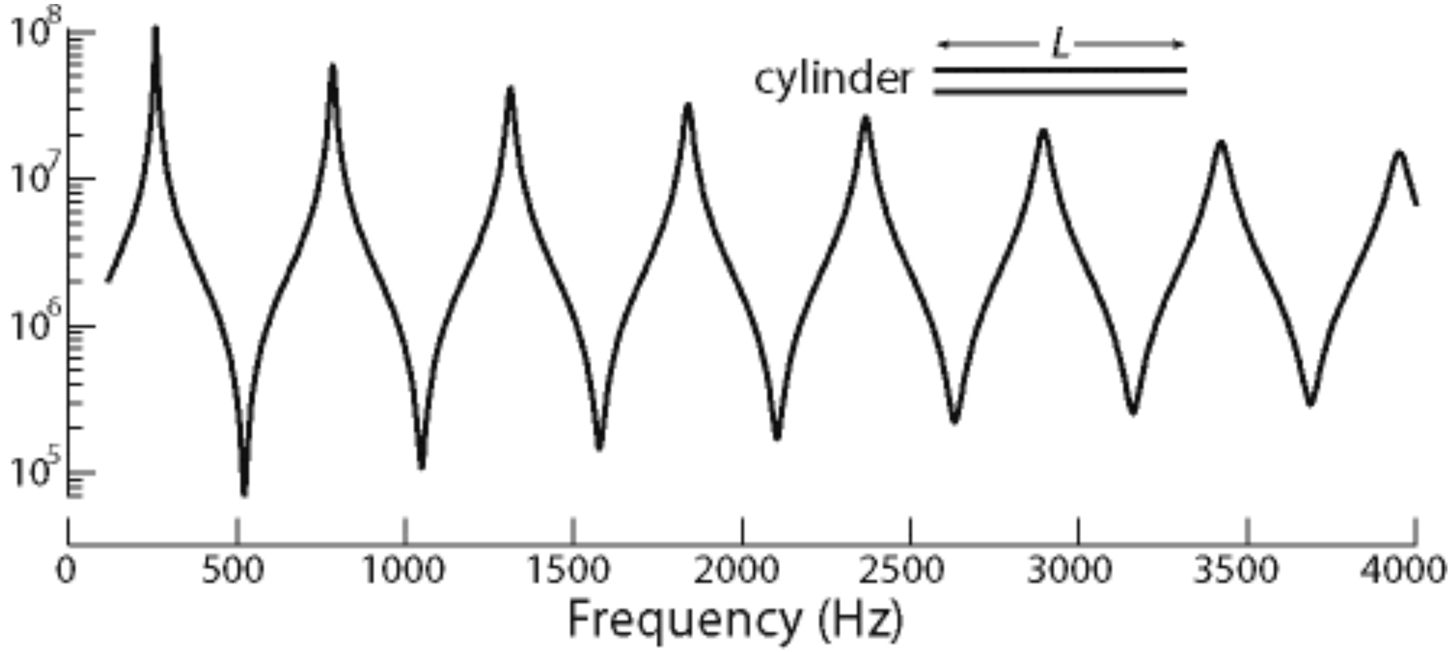
- Impedance is a useful concept in mechanical, electrical, and acoustical systems

System	**Effort Variable**	**Flow Variable**	**Ohm's Law**
Electrical	voltage V	current I	$\Delta V = IZ$
Acoustic	pressure p	flow rate U	$\Delta p = UZ$
Mechanical (trans.)	force F	velocity v	$\Delta F = vZ$
Mechanical (rot.)	torque τ	ang. vel. ω	$\Delta \tau = \omega Z$

- Translation and rotation: pressure/voltage are like force/torque, and current is like velocity

Impedance of a Tube of Air

- Impedance *Z*(*f*) of a finite cylinder:

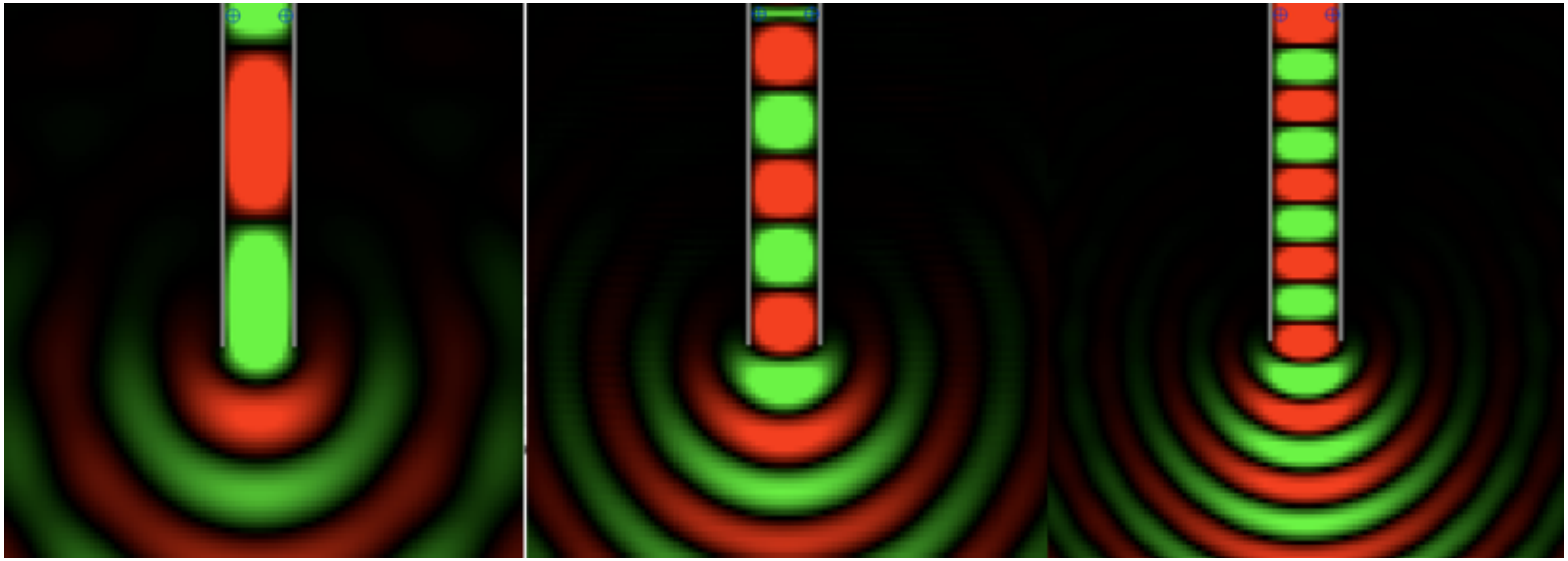


J. Wolfe, UNSW

- Varying pressure at input pushes a wave down the tube
- The wave reflects at the end of the tube and comes back, producing standing waves (resonances)
- Impedance is high when returning wave is in phase with driving pressure, and low when it's out of phase

Reflections at the End

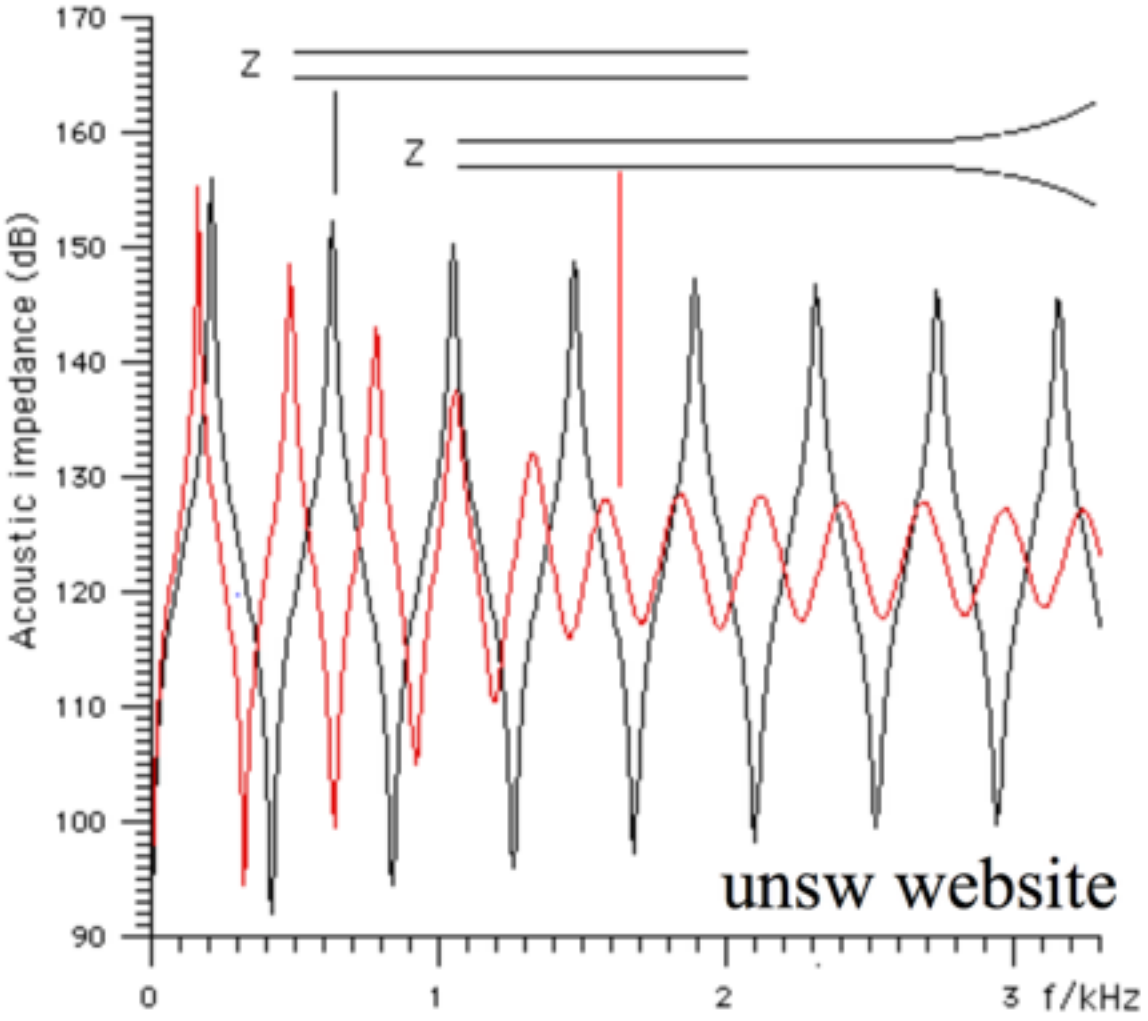
- Ripple tank simulations (falstad.com)



- Low frequency waves are reflected at end of tube
- High frequency waves are more efficiently radiated

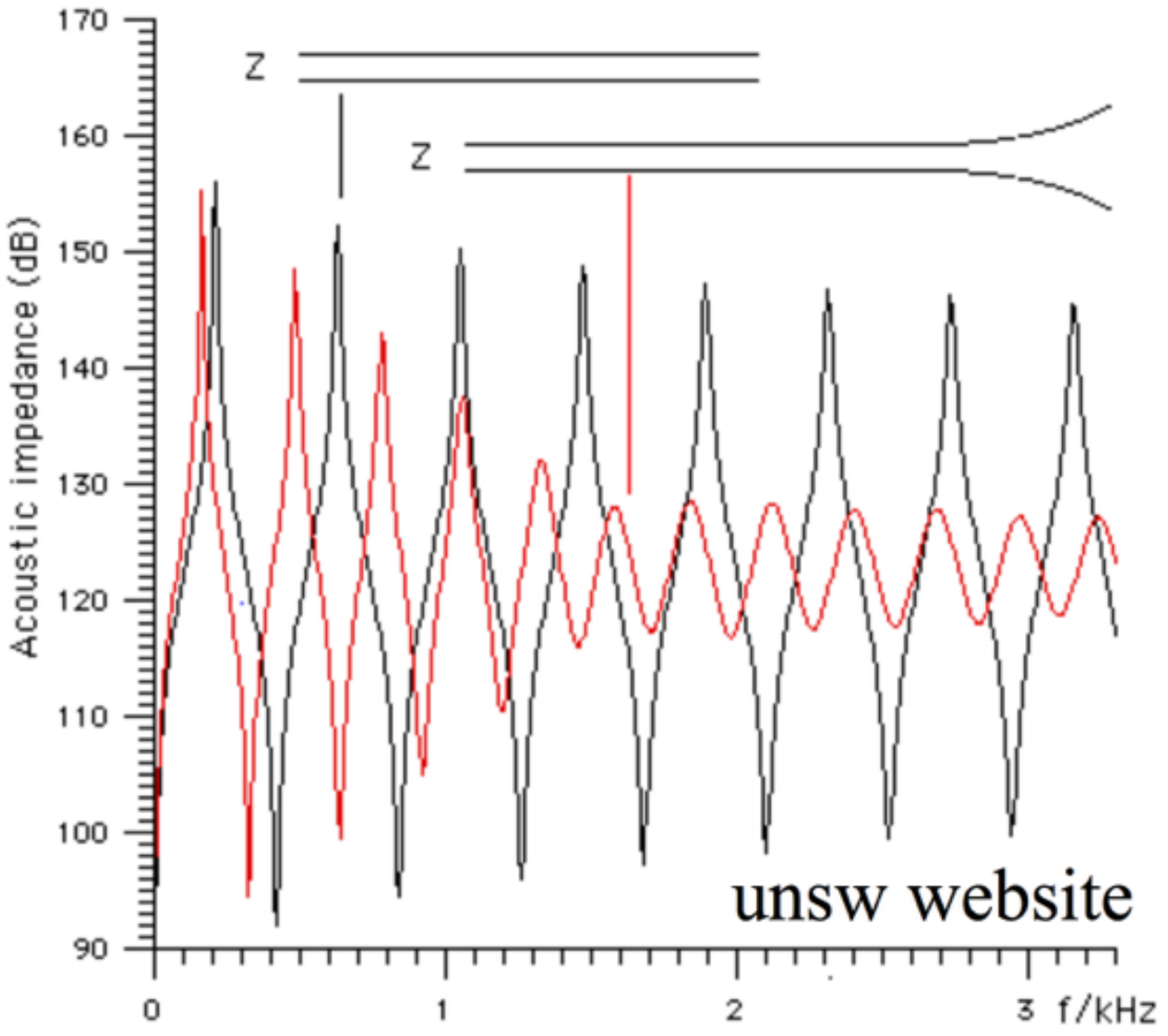
Impedance Matching at Bell

- Low frequency, large wavelength pressure waves reflect at far end of the tube
- The bell radiates high frequencies more efficiently (less reflection)
- Result: impedance is lowered at high *f*, leading to weaker resonances
- Also note shift in harmonics!



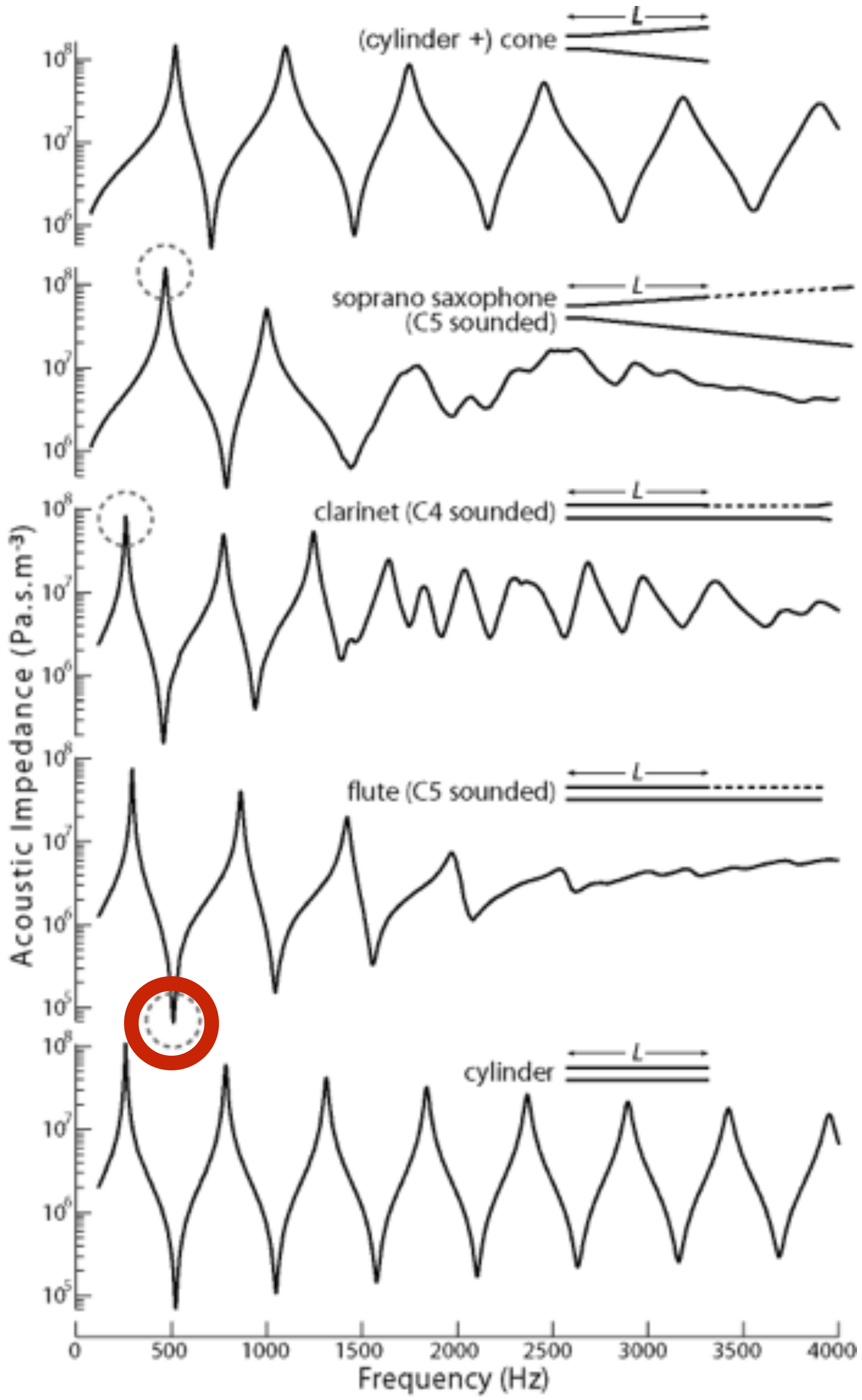
unsw website

Effect of Bore Shape



- Listen to a tube played with and without the bell

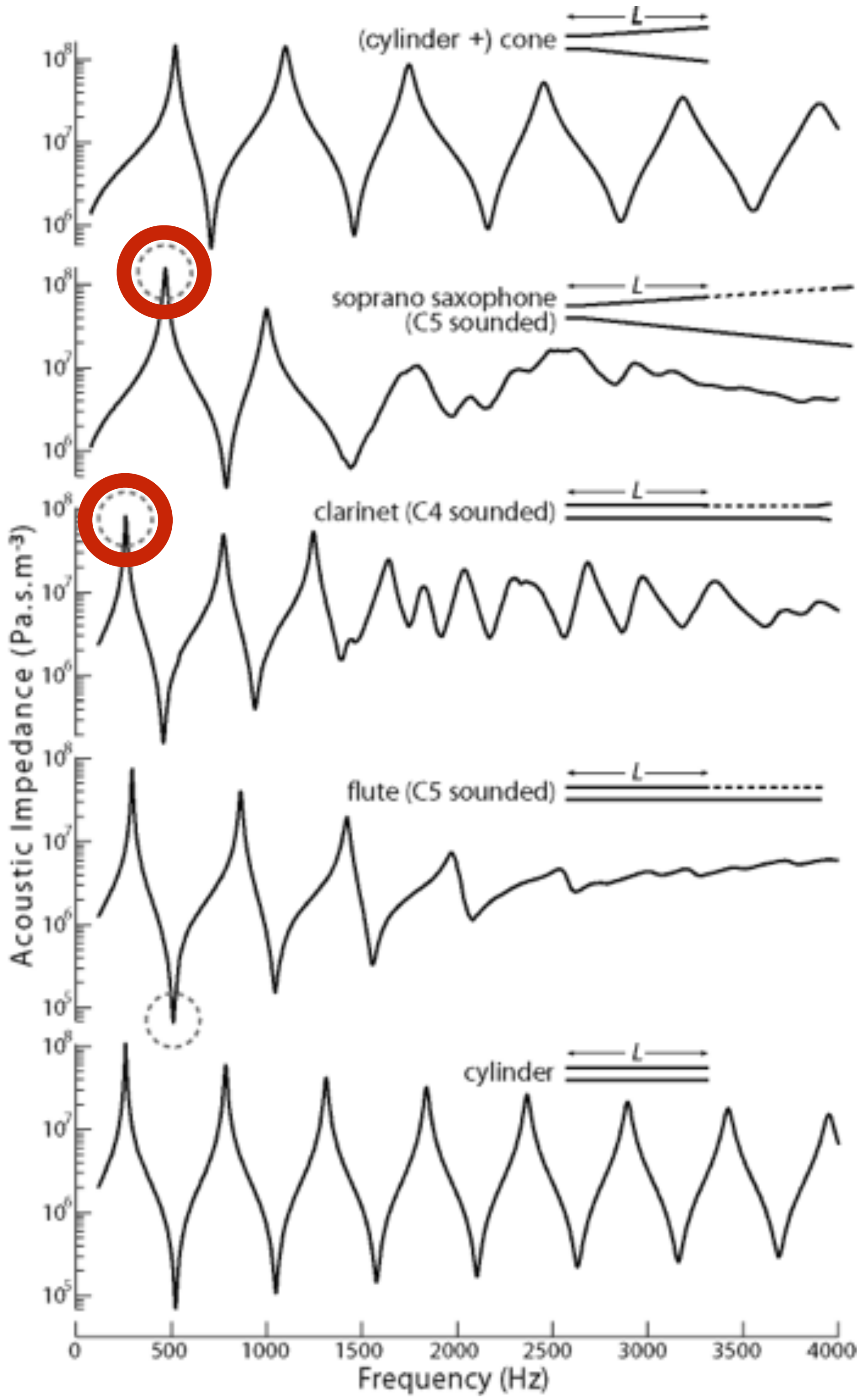
Location of Resonances



J. Chen et al., Acoustics Australia 37:18-23, 2009

- Flute: open-open tube, mouthpiece is open to atmosphere
- Acoustic pressure is ~0 at mouthpiece
- Oscillating air flow is large at the mouthpiece (recall phase difference between displacement and pressure)
- Resonances occur at low impedance points

Location of Resonances

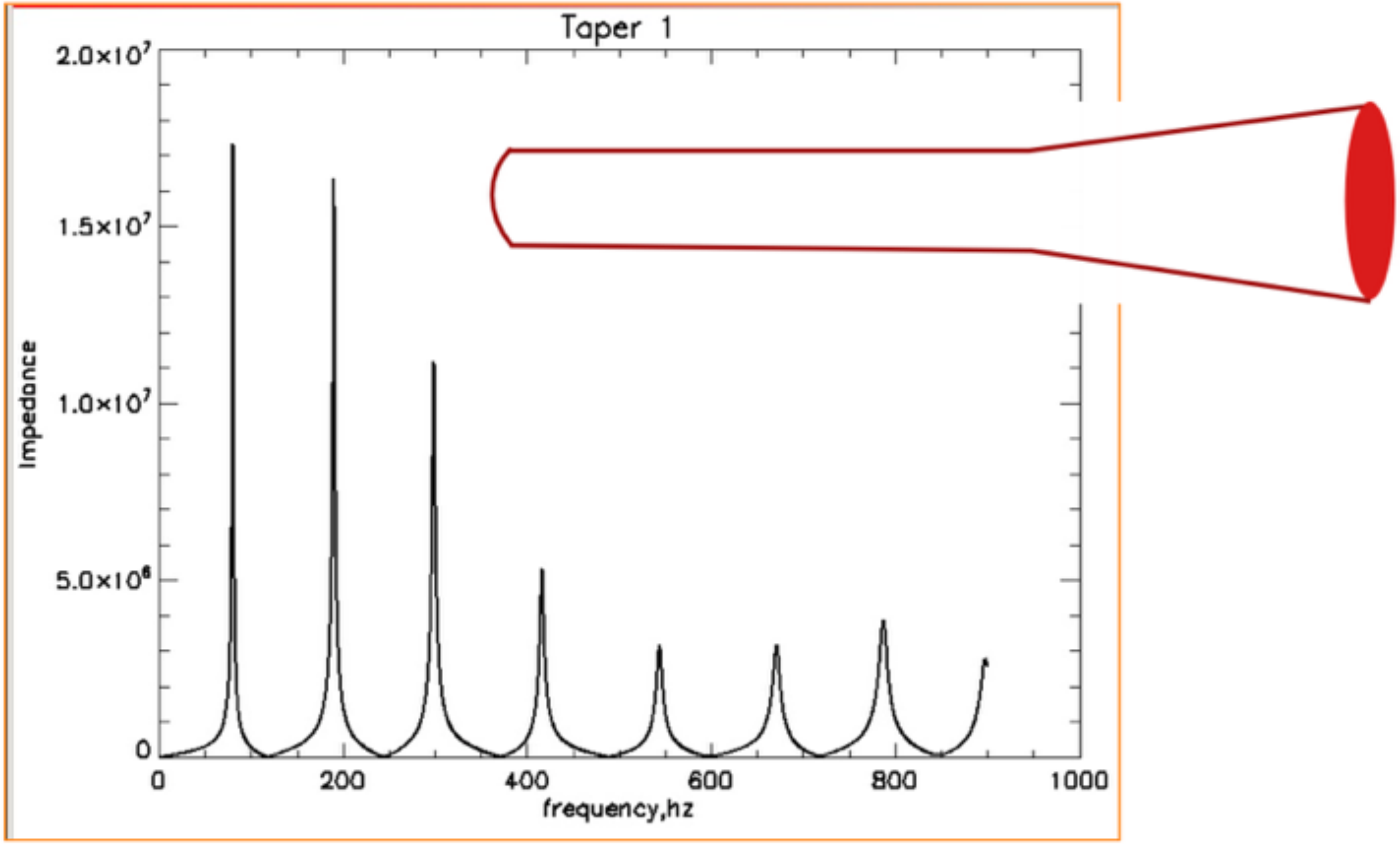


J. Chen et al., Acoustics Australia 37:18-23, 2009

- Clarinet, saxophone: closed-open tube
- Acoustic pressure is large at mouthpiece
- Oscillating air flow is small at the reed (closed end)
- Resonances occur at high impedance points

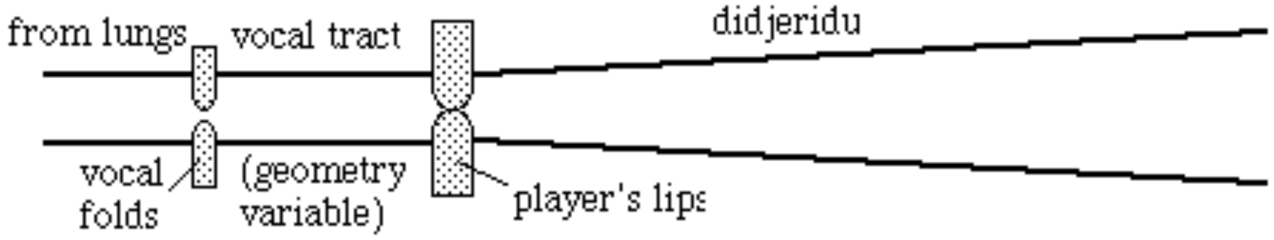
Impedance of Didgeridoo

- The weaker the peak in the impedance plot, the more easily sound is lost to the room



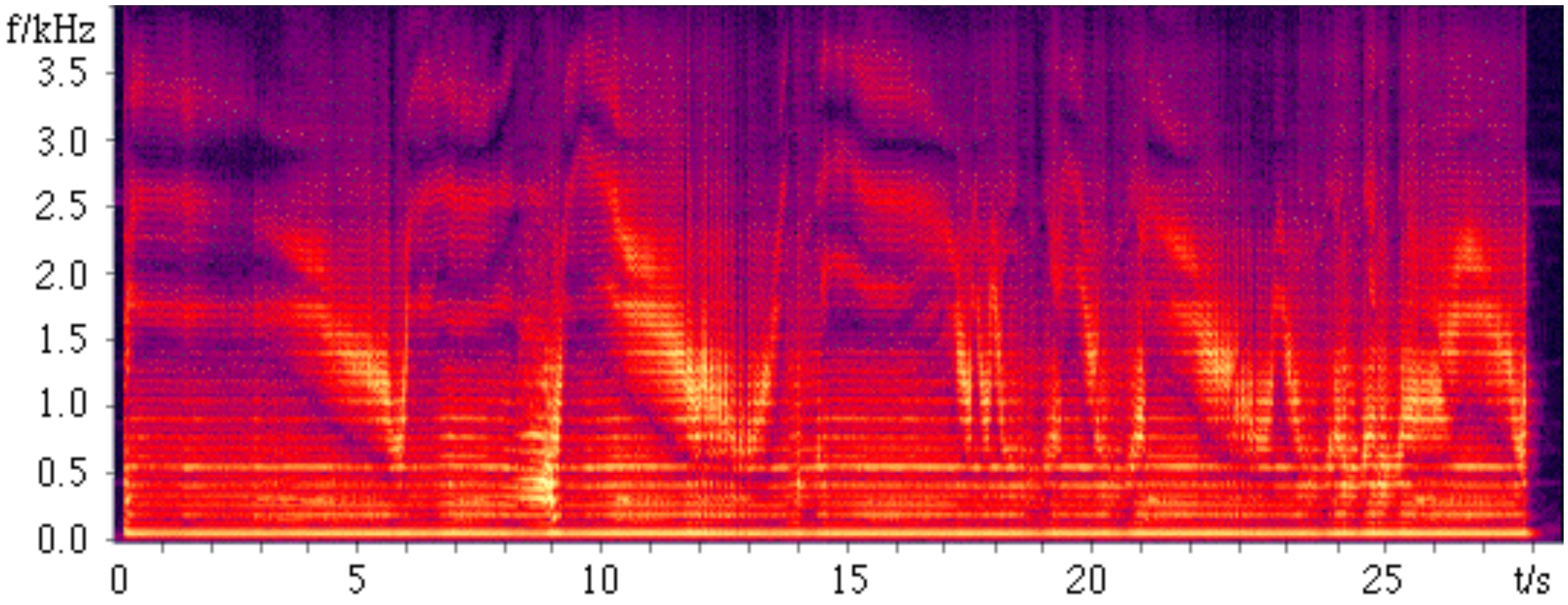
Effect of Vocal Tract

- The lips produce a sound wave that travels into the instrument, but the sound also travels into the vocal tract, which acts like a resonator



David Finlayson, Peter Ellefson, Jay Bulen

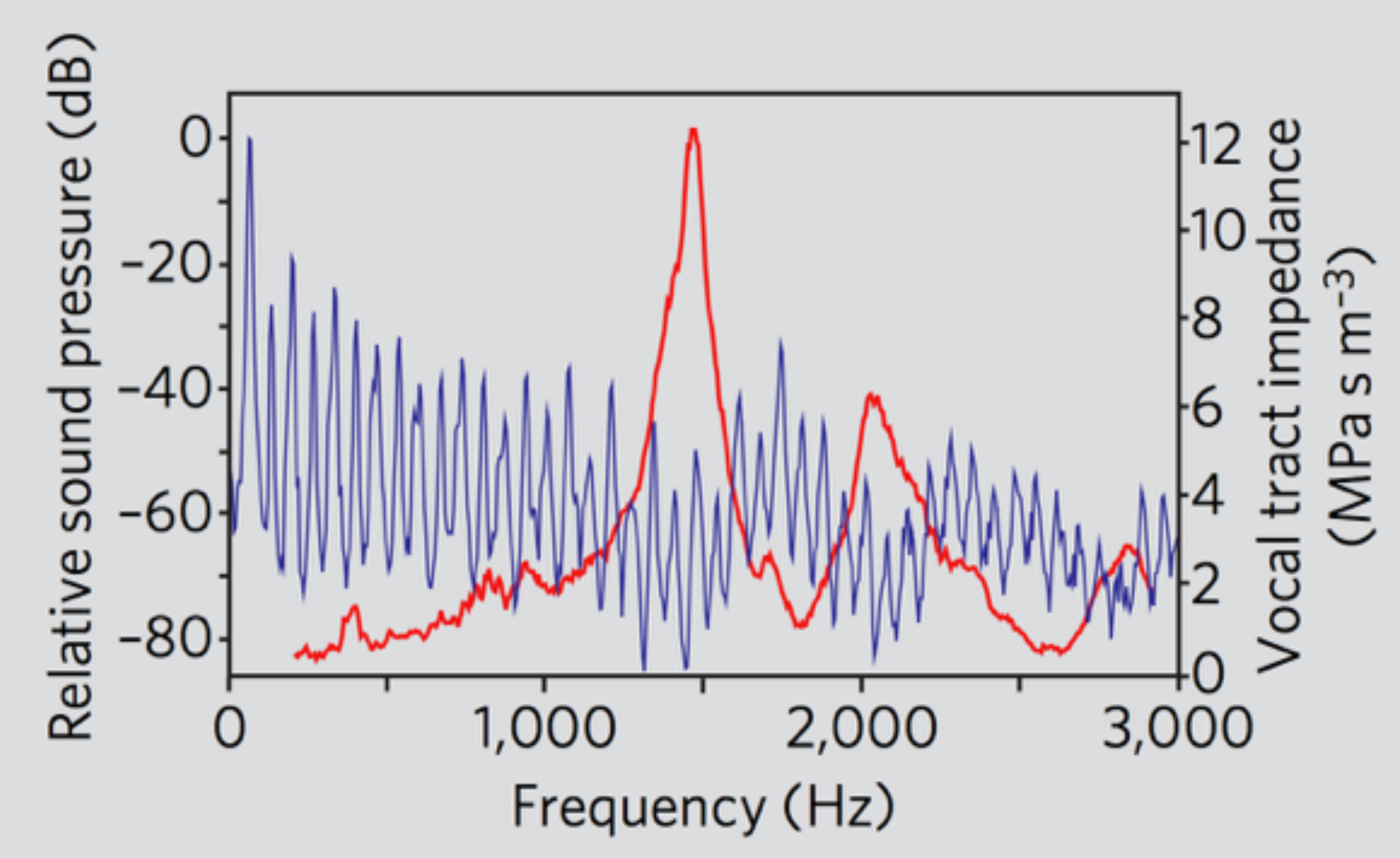
J.Wolfe, UNSW



Vocal Tract Impedance

- Didgeridoo players can change the impedance of their vocal tract altering the shape of the vocal cavity

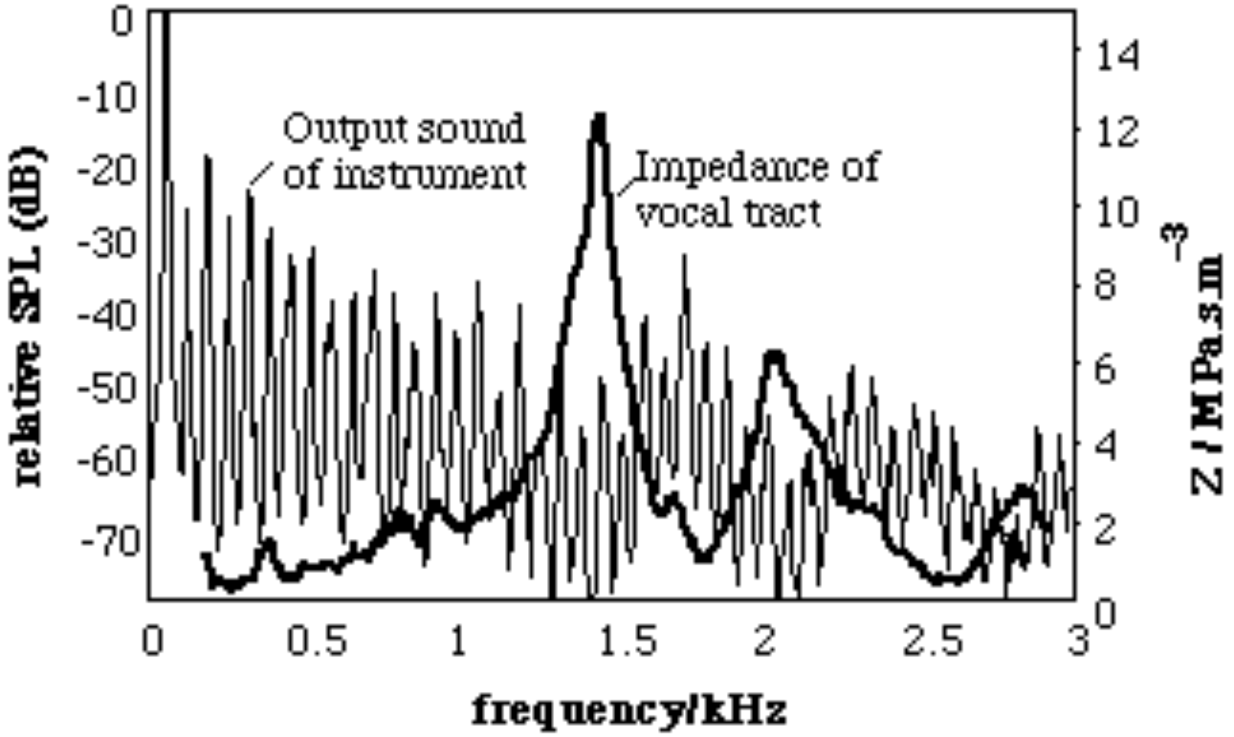
A. Tarnopolsky et al., Nature 436:39, 2005



- Produces regions of heightened response, known as formants. Note: this is how we produce vowel sounds

Didgeridoo Output

- Effect of formants of the vocal tract on the output of the didgeridoo



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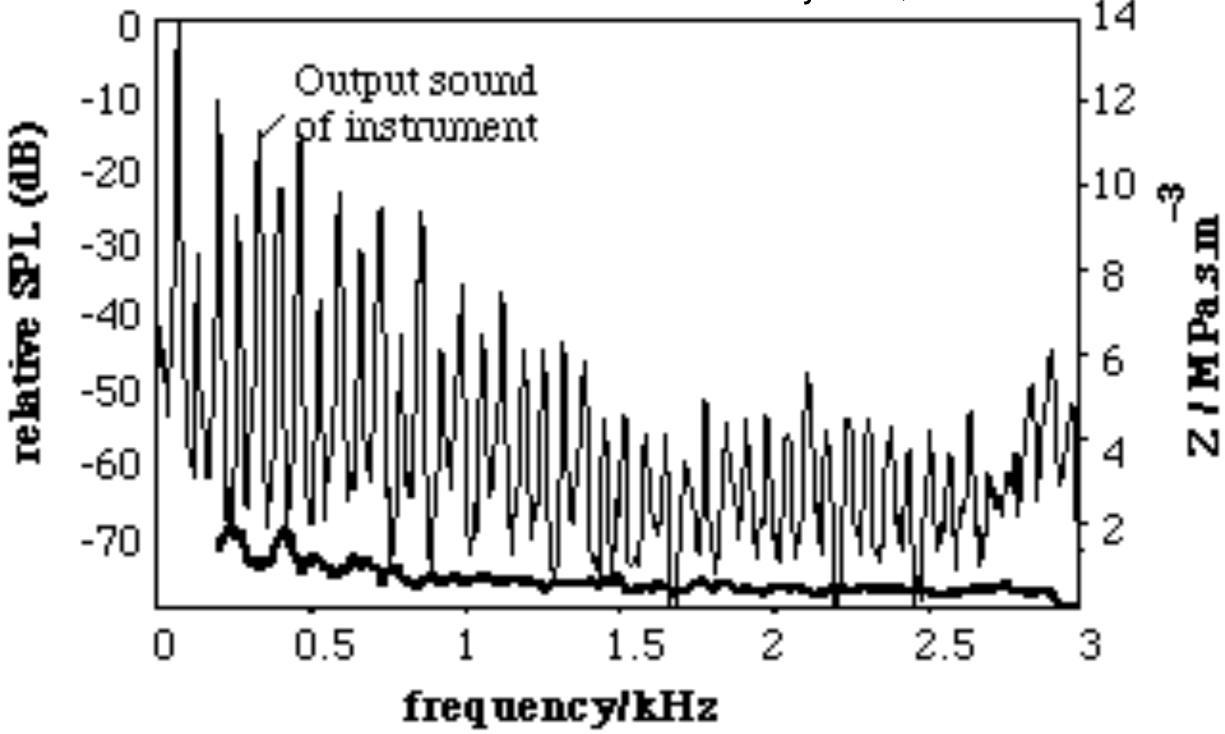

- Left: tongue placed high in the mouth near the hard palate. Right: tongue placed low in the mouth

Measuring Impedance

- If we drive an open-open tube with a noise source (white noise), frequencies at low impedance will be amplified by the tube
  - Instant measurement of $Z(f)$ for all f!
  - Not a very accurate measurement
- Better: use a forced oscillating air flow source at constant amplitude
  - Measure pressure variations caused by source
  - Scan through f to determine $Z(f)$

Summary

- Impedance measures how effectively energy is transferred from one body to another
- In acoustics, impedance (Z=pressure/flow) tells us how easy it is to play certain sounds in an instrument
- Matching impedances between different elements in an instrument is critical for design
  - Couple low impedance elements (e.g., a string) to high impedance elements (violin body) with intermediate impedance elements (bridge)
  - Impedance matches are usually desired, but sometimes accidental, as in the case of Wolf notes