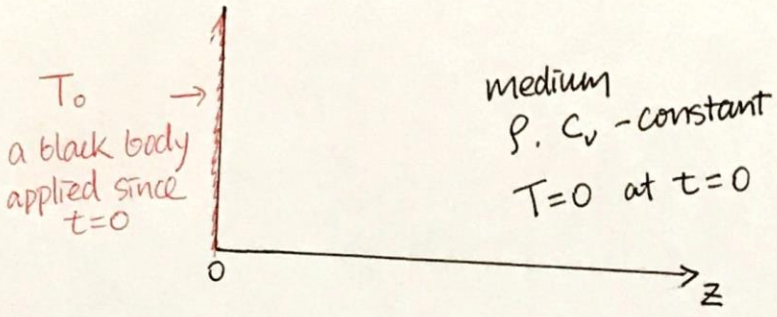
Marshak wave

Castor § 12.1

a thermal diffusion



$C_v \sim$ const.

$\kappa_R \propto T^{-n}$

$d \propto t^{1/2}$

$$F = -\frac{16}{3}\frac{\sigma T^3}{\kappa_R \rho}\frac{\partial T}{\partial z} \qquad (1)$$

$$\kappa_R = \overbrace{\kappa_R(T_0)}\left(\frac{T}{T_0}\right)^{-n} \qquad (2)$$

$n=0$ e^- scattering

$n=3$ bound-free, free-free absorption

(2)→(1):

$$F = -\frac{16}{3}\frac{\sigma T^3}{\overbrace{\kappa_R(T_0)}}\left(\frac{T}{T_0}\right)^n \frac{1}{\rho}\frac{\partial T}{\partial z}$$

$$= -\frac{16}{3}\frac{\sigma}{(n+4)T_0^{\,n}\overbrace{\kappa_R(T_0)}\rho}\frac{\partial T^{n+4}}{\partial z} \qquad (3)$$

diffusion

$$\rho c_v \frac{\partial T}{\partial t} = -\frac{\partial F}{\partial z}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_v}\frac{\partial}{\partial z}\left(\frac{16\sigma}{3(n+4)T_0^{\,n}\overbrace{\kappa_R(T_0)}\rho}\frac{\partial T^{n+4}}{\partial z}\right) \qquad (4)$$

$$g \equiv \frac{T}{T_0} \qquad (5)$$

$$\xi = \frac{K}{\sqrt{t}}z \qquad (6)$$

$$K = \left(\frac{3(n+4)\overbrace{\kappa_R(T_0)}\rho^2 c_v}{32\sigma T_0^{\,3}}\right)^{1/2} \qquad (8)$$

$\Rightarrow$ equation to be solved

$$-\xi \frac{dg}{d\xi} = \frac{d^2 g^{n+4}}{d\xi^2} \qquad (7)$$

$$g \sim \left[\frac{(n+3)\,\xi_{max}}{n+4} \left(\xi_{max} - \xi\right) \right]^{1/(n+3)} \qquad (10)$$