1. MHD eq.'s including radiation

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\vec{v}) \quad ①$$

$$\frac{\partial \vec{p}}{\partial t} = -\nabla\cdot(\vec{v}\otimes\vec{p} - \tau) - \nabla P + \vec{J}\times\vec{B} + \rho\vec{g} - \nabla\overleftrightarrow{P}_{rad} \quad ②$$

$$\frac{\partial e}{\partial t} = -\nabla\cdot(e\vec{v}) - P\nabla\cdot\vec{v} + Q + Q_{rad} \quad ③$$

$\vec{p}$ - the momentum density, τ - the stress tensor

P - the gass pressure, $\vec{J}$ - the current density

$\vec{B}$ - the magnetic field vector, $\vec{g}$ - the acceleration due to gravit

$\overleftrightarrow{P}_{rad}$ - the radiation pressure tensor, e - the internal energy

Q_{rad} - heating/cooling due to rad., Q - other energy exchange

2. Energy density of radiation and matter

$T_{rad} = 5777$ K

$E_{rad} = \frac{4\sigma}{c} T_{rad}^4 = 0.84$ Jm^{-3} ④

assume B.B:

$F_{rad} = \sigma T_{rad}^4 = 6.3 \times 10^7$ Wm^{-2} ⑤

3. Radiation pressure and force

Isotropic B.B rad.

$\frac{4\sigma}{3c} T^4 = 0.27$ Pa @ $T_{ph} = 5700$ K ⑥

$P_{gas} \sim 10^4$ Pa

$a_{rad} \approx \frac{\kappa F}{c} = 3 \times 10^{-3}$ ms^{-2} ⑦

κ - the Rosseland ~~mean~~ opacity, F - the freq.-integrated rad. flux

$g_{\odot, surf} \sim 274$ ms^{-2}

4. Energy exchange between radiation and matter

energy gaining rate per volume, by a medium from rad. field

$$Q_{rad} = -\int_0^\infty \nabla \cdot \vec{F}_\nu \, d\nu = -\nabla \cdot \vec{F} \quad (8)$$

$\vec{F}$ - the total radiative flux

5. Explicit expression of the

$$\vec{F}_\nu = \oint \hat{n} I_\nu \, d\Omega \quad (9)$$

source function: $S_\nu = j_\nu / (\kappa_\nu \rho)$

$$\begin{aligned} \nabla \cdot \vec{F} &= \int_0^\infty \oint (j_\nu - \kappa_\nu \rho I_\nu) \, d\Omega \, d\nu \quad (10) \\ &= \int_0^\infty \oint \kappa_\nu \rho (S_\nu - I_\nu) \, d\Omega \, d\nu \end{aligned}$$

If both S_ν and κ_ν do not depend on direction:

$$\nabla \cdot \vec{F} = \int_0^\infty 4\pi \kappa_\nu \rho (S_\nu - J_\nu) \, d\nu \quad (11)$$

the angle-averaged intensity:

$$J_\nu = \frac{1}{4\pi} \oint I_\nu \, d\Omega \quad (12)$$

6. Light travel time and hydrodynamical timescales

$v_{bulk,\, solar\, atm}$ $1 \sim > 300$ km s^{-1}

$v_{Alfvén}$ up to 2.2×10^3 km s^{-1}

t_{hydro} ~ 5 min in photosphere

~ 1 min in chromosphere

few min in corona

$t_{light\, cross}$ ~ 1 sec

t-dependent transfer eq.

$$\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{n} \cdot \nabla I_\nu = j_\nu - \alpha_\nu I_\nu \quad (13)$$

simplifies to $\Rightarrow$

$$\hat{n} \cdot \nabla I_\nu = j_\nu - \alpha_\nu I_\nu \quad (14)$$

j_ν - emissivity , $\alpha_\nu = \kappa_\nu \rho$ extinction coeff. per volume

7. Diffusion approximation

At very large optical depth

$$J_\nu \approx S_\nu + \frac{1}{3}\frac{d^2 S_\nu}{d\tau_\nu^2} \qquad (15)$$

$$\vec{F}_\nu \approx -\frac{4}{3\kappa_\nu \rho}\nabla S_\nu \qquad (16)$$

for all freq. $\Rightarrow$
assume $S_\nu = B_\nu$

$$\vec{F} \approx -\frac{16}{3}\frac{\sigma T^3}{\kappa_R \rho}\nabla T \qquad (17)$$

Rosseland opacity

$$\frac{1}{\kappa_R} = \left(\int_0^\infty \frac{1}{\kappa_\nu}\frac{dB_\nu}{dT}\,d\nu\right) \Big/ \left(\int_0^\infty \frac{dB_\nu}{dT}\,d\nu\right) \qquad (18)$$