Today in Astronomy 142: stellar-mass black holes

Figure: artist’s conception of a star - black hole binary system (Dana Berry, Honeywell/NASA.)
Escape speeds from stars, white dwarfs and neutron stars

Neglecting relativity, in increasingly-bad approximation:

\[
E = \frac{1}{2} mv_{\text{esc}}^2 - \frac{GMm}{R} = 0
\]

\[
v_{\text{esc}} = \sqrt{\frac{2GM}{R}}
\]

- \(v_{\text{esc}} = 619 \text{ km s}^{-1} = 0.002c\) (Sun)
- \(v_{\text{esc}} = 6970 \text{ km s}^{-1} = 0.023c\) (1\(M_\odot\) white dwarf)
- \(v_{\text{esc}} = 154000 \text{ km s}^{-1} = 0.514c\) (1\(M_\odot\) neutron star)

\[
v_{\text{esc}} = c = 299792 \text{ km s}^{-1} \text{ when } R = 2GM / c^2
\]
Beyond the neutron-star maximum mass: black holes

- The maximum mass of a neutron star is about $2.2 \, M_\odot$. There is no physical process that can support a heavier object without internal energy generation.

- A (non-spinning) heavier object will collapse past neutron-star dimensions, and soon thereafter becomes a **black hole**: an object from which even light cannot escape if emitted within

  $$R_{\text{Sch}} = \frac{2GM}{c^2}$$

  *Schwarzschild radius*

  of the object, as measured by a distant observer; this spherical surface is called the **event horizon**, or simply the “horizon.”
Black holes and general relativity

- The “nonrelativistic” result (page 2) turns out accidentally to be the same as that obtained with the general theory of relativity (GR), as you will learn in AST 231.

- GR is a description of the effect of gravity at any strength – even large amounts of mass, shrunk to small dimensions – and in turn involves new and different mathematical tools we can’t attempt to introduce in AST 142.

English translation of the main tools, which in AST 231 you will learn as the Einstein field equations:

- Masses cause space and time to be curved, or warped.
- The resulting curvature of space and time determine how masses will move.

We’ll content ourselves here with a list of the salient facts…
Interesting facts about black holes

Time and space are warped substantially near black holes.

- Time intervals on clocks near black holes appear to distant observers to be slow compared to their (identical) clocks, an effect known as **gravitational time dilation** or the **gravitational redshift**:

  
  \[ \Delta t = \Delta \tau / \sqrt{1 - R_{\text{Sch}} / r} > \Delta \tau \]

- Thus **time appears (to a distant observer) to stop at the event horizon**:

  \[ \Delta t \rightarrow \infty \text{ as } r \rightarrow R_{\text{Sch}} \]

This behavior gave the horizon its original name: “Schwarzschild singularity.”
Interesting facts about black holes (continued)

- Near a black hole, a (small) length $\Delta L$ measured \textbf{instantaneously} (as with a measuring tape) between two points on a \textit{radial} line is greater than the distance $\Delta r$ between the points, measured by a distant observer:

$$\Delta L = \frac{\Delta r}{\sqrt{1 - \frac{R_{\text{Sch}}}{r}}} > \Delta r$$

- Neither $\Delta r$, nor the distant-observer-measured radius $r$ of a point near a BH, has meaning as a physical distance to an observer near a black hole.
  - $r$ and $\Delta r$ are called \textbf{coordinate distances}.

- However, the \textbf{circumference} $C$ of the circle through that point, centered on the BH, \textbf{turns out to have the same value in all frames}. Think of $r$ only as $r = \frac{C}{2\pi}$. 


Circular orbits and their radii in GR

Circles in flat spacetime: $C = 2\pi r = \pi d$. That, of course, is the very definition of $\pi$. 
Circular orbits in flat space (all in the same plane)

Distances between the orbits:

Local and distant observers would report the same distances.
Circular orbits in space warped by a black hole, same circumferences as before (still all in same plane)

Distances between the orbits, measured by a local observer:

1.057  1.074  1.185
1.065  1.087  1.135  1.300

The distances would still all be 1, in the viewpoint of a distant observer.
Circumferences are the same in both viewpoints.

Horizon (circumference = 2\pi)
One way to visualize warped space: “hyperspace”

To connect these circles with segments of these “too long” lengths, one can consider them to be offset from one another along some imaginary dimension that is perpendicular to $x$ and $y$ but is not $z$. (If it were $z$, the circles wouldn’t appear to lie in a plane.) Such additional dimensions comprise hyperspace.
This is why you often see the equatorial plane of a black hole represented as a funnel-shaped surface, as if made from a stretched rubber sheet. It’s important to note that the direction of the stretch is in hyperspace, though, so the scene would not look like a funnel to your eyes, which see just the three usual spatial dimensions.
Interesting facts about black holes (continued)

- Orbits outside the BH’s horizon, further away than 1.5 $R_{Sch}$ (in the coordinate system of a distant observer), still turn out to be ellipses.

- The resulting **coordinate speed** in orbit (for the coordinate system of a distant observer) is the same as is obtained for Newtonian gravity:

  $$v \equiv r \frac{d\phi}{dt} = \sqrt{\frac{GM}{r}}$$

- At the horizon, the radial component of the coordinate speed of light is zero: **light cannot escape**. Thus no information can reach a distant observer from, or within, the horizon.
Interesting facts about black holes (continued)

- For non-spinning black holes, orbits with coordinate radius $< 3 R_{\text{Sch}}$ are unstable to small perturbations.
- There are no orbits with coordinate radius $< 1.5 R_{\text{Sch}}$ for a non-spinning black hole; at this radius the local orbital speed is the speed of light, and smaller orbits would require (impossibly) higher speeds.
  - Thus you can’t orbit there, because your rest mass isn’t zero, but if you could, you could train your binoculars straight ahead (in the $\phi$ direction) and see the back of your head.
  - To get closer to the horizon, one would have to descend radially and balance gravity with thrust, as in a rocket launch.
Interesting facts about black holes (continued)

- If the black hole spins, the innermost stable orbit and the photon orbit are smaller than 3 and 1.5 $R_{Sch}$ if the particle orbits in the same direction as the spin, and larger if it orbits in the opposite direction.

- Within $r = 1.5 R_{Sch}$ all geodesics (possible paths for light) terminate at the horizon.

- Thus: from near the horizon, the sky appears to be compressed into a small range of angles directly overhead; the range of angles is smaller the closer one is to the horizon, and vanishes at the horizon.
  - The objects in the sky appear bluer than their natural colors as well, because of the gravitational Doppler shift.
Interesting facts about black holes (continued)

- Space itself is stuck to the horizon, since one end of all the geodesics are there. If the horizon began to rotate, the ends of the geodesics would rotate with it. (This harmonizes with time stopping there.)

- Gravitational acceleration turns out to be

\[ a = \frac{GM}{r^2} \frac{1}{\sqrt{1 - R_{Sch}/r}} \]

which has its familiar Newtonian form at large distances but blows up at \( r = R_{Sch} \). Thus, in a vertical descent to a hovering position just above the horizon, very large gravitational accelerations would be encountered.
Interesting facts about black holes (continued)

- Tidal forces turn out the same near a black hole as in Newtonian gravity, and are finite at the horizon. For an object of length $\Delta r$ in the radial direction and $\Delta x$ in the crosswise directions,

$$\Delta a_r = \frac{2GM}{r^3} \Delta r \quad \Delta a_\phi = -\frac{GM}{r^3} \Delta x$$

- For a 2 m person and a $10 M_\odot$ BH, the radial tidal acceleration $\Delta a_r$ at the event horizon is $2 \times 10^{10}$ cm sec$^{-2}$ ($2 \times 10^7 g$).

- $\Delta a_r = 1g$ for a $4.6 \times 10^4 M_\odot$ BH. Thus if you want to fall freely past the horizon of a BH to see what happens, choose a large one, so as not to be torn apart before you get there.
Yet black holes emit light!

Details of the process, called **Hawking radiation**:

- Virtual particle-antiparticle pairs, produced briefly by vacuum fluctuations, can be split up by the strong gravity near a horizon. Both of the particles can fall in, but it is possible for one to fall in with the other escaping.

- The escaping particle is seen by a distant observer as emission by the black hole horizon: black holes emit light (and other particles)!

- The energy conservation debt involved in the un-recombined vacuum fluctuation is paid by the black hole itself: the black hole’s mass decreases by the energy of the escaping particle, divided by $c^2$. The emission of light (or any other particle) costs the black hole mass and energy.
Black hole evaporation

- Hawking radiation is emitted more efficiently if the tides at the horizon are stronger. You will show in workshop this week that tides at the horizon are larger for smaller-mass black holes.

- Emission is the same as that of a blackbody at temperature

\[ T = \frac{hc^3}{16\pi^2kGM} \left( \propto \sqrt{\Delta a_r} \right) \]

- Thus an isolated black hole will eventually evaporate, as you’ll show in Homework #4.
  
  - Evaporation takes this long –
    - \(10^9 M_\odot\) black hole: \(10^{94}\) years.
    - \(2 M_\odot\) black hole: \(10^{67}\) years.
    - \(10^8\) gram black hole: 1 second (!)
“Black holes have no hair”

Meaning: after collapse is over with, the black hole horizon is smooth: nothing protrudes from it; and that almost everything about the star that gave rise to it has lost its identity during the black hole’s formation. No “hair” is left to “stick out.”

- Any protrusion, prominence or other departure from spherical smoothness gets turned into gravitational radiation; it is radiated away during the collapse.
- Any magnetic field lines emanating from the star close up and get radiated away (in the form of light) during the collapse.
- The identity of the matter that made up the star is lost. Nothing about its previous configuration can be reconstructed.
“Black holes have no hair” (continued)

- Even the distinction between matter and antimatter is lost: two stars of the same mass, but one made of matter and one made of antimatter, would produce identical black holes.

The black hole has only three quantities in common with the star that collapsed to create it: **mass, spin and electric charge.**

- That is, in common with the star as it was immediately before the formation of the horizon...

- Only very tiny black holes can have much electric charge; stars are electrically neutral, with equal numbers of positively- and negatively-charged elementary particles.

- Spin makes the black hole depart from spherical shape, but it’s still smooth.
Real stellar-mass black holes: GRO J1655-40

We know of ~40 good candidates for stellar-mass black holes, of which at least 20 are rock solid. Here’s a good example of the evidence that comprises a rock-solid case.

GRO J1655-40 is a bright, soft X-ray transient (burst) object discovered by the Compton GRO in 1994.

- Appeared as a nova (= Nova Scorpii 1994) in visible light, during the X-ray burst.
- Suffered another nova/X-ray outburst in 2005.
- Normal stars (unless very young), and ordinary novae, do not emit much light at X-ray wavelengths.
  - To get electric charges to emit X-rays, one has to accelerate them close to $c$, which if done with gravity would require an neutron star or a black hole.
When bursting, GRO J1655-40 exhibits rapidly-variable X-ray emission: \(\times 2\) brightness changes in \(\Delta t \approx 3 \times 10^{-4}\) sec.
- So the object is at most a hundred km across, for which the light travel time would be about that.
- And that is way too small to be a star or a white dwarf.

When not bursting, it looks like a normal star, rather similar to the Sun (V1033 Sco, \(m_1 = 1.1M_\odot\)).
- Star’s brightness indicates a distance of 3200 pc.

The star’s spectral lines show it to be a single-line spectroscopic binary system: star and invisible companion in orbit.
- We can presume that the X-ray-bright object is the invisible companion.
GRO J1655-40 (continued)

- Period $P = 2.62$ days, velocity amplitude = 216 km s$^{-1}$.
- Thus the mass function (Homework #1) is

$$f(m_1, m_2) = \frac{P \nu_1 r}{2\pi G} = 2.7M_\odot < m_2$$

- The star is eclipsed when the system is in outburst, but not when it is quiescent, so we view the orbit close to edge-on but not exactly: $70^\circ$ from the line of sight.
- And thus we know the mass of the X-ray bright companion (and the star) rather precisely (Shahbaz 2003):

$$m_2 = 5.99 \pm 0.42M_\odot$$

Companion definitely exceeds maximum neutron-star mass.
GRO J1655-40 also emits relativistic bipolar jets, for which the speed and orientation can be measured from the proper motion of the clumps in the jets (Hjellming & Rupen 1995).

- Outflow speed is 0.92c; orientation is 85° from the line of sight, 15° from the system rotation axis.
- The shapes of the clumps indicate that the jet is precessing with a period similar to the orbit.

Ejection speeds tend to be similar to escape speeds. Nothing besides a black hole would eject material at 0.92c.
A 6 $M_\odot$ nonspinning black hole has a horizon circumference 111 km, and an innermost stable orbit of 333 km. Material in this orbit will circle the black hole **367 times per second**.

- However, one often sees the X-ray brightness of GRO J1655-40 modulate at **450 times per second** for long stretches of time ([Strohmayer 2001](#)).
  - This behavior is called **quasiperiodic oscillation**.

- Nothing besides very hot material in a stable orbit can do this so reproducibly at this frequency.

- Thus there are stable orbits closer to the black hole than they can be if it doesn’t spin.

- The maximum spin rate of a BH corresponds to a coordinate speed at the horizon of $c$. GRO J1655-40 spins at 21% of this rate.
In **blue**: innermost stable orbital frequency for $5.99 \pm 0.42 M_\odot$ black holes, as is measured for GRO J1655-40.

In **red**: frequency of the quasiperiodic oscillations in GRO J1655-40 (Strohmayer 2001).
GRO J1655-40 (continued)

Thus we have many lines of evidence that GRO J1655-40 is a binary system consisting of a main-sequence $1.1M_\odot$ star and a $6.0M_\odot$ black hole. This is as solid as such a case gets:

- High-energy radiation (X-rays).
- “Variability size”: X-ray object too small to be a star.
- Orbital dynamics: mass of dark companion is precisely determined, and is far too large to be a neutron star or white dwarf.
  - And it’s way too faint to be a $6M_\odot$ star, of course.
- Quasiperiodic X-ray oscillations easily explained as due to spin in a BH at the same mass determined above.
- Relativistic jets: ejected from system at nearly light speed.
Here’s the picture of GRO J1655-40 which emerges from these results.

Note:

- Like GRO J1655-40, all the other stellar-mass black holes we know (~40 to date) belong to X-ray emitting binary systems.

- Without X-ray emission from material on its way to being accreted, they would be inconspicuous.
Summary of $C$ vs. $M$ for degenerate stars

- White dwarfs
- Neutron stars
- Black holes
- Earth
- Rochester

No stable objects below the EOS causality limit (—).