Today in Astronomy 142: Ever Since Decoupling

- Anisotropies in the cosmic background radiation, and a new standard ruler.
- The Universe is flat.
- The models fit but problems remain: topics for the next generation of Ph.D. theses in cosmology.

The bad news

Exam #2 takes place here, on Tuesday.

- To the test bring only a writing instrument, a calculator, and one 8.5"×11" sheet on which you have written all the formulas and constants that you want to have at hand.
  - No computers, no access to internet or to electronic notes.

- The best way to study is to work problems like those in homework and recitation, understand the solutions and reviews we distributed, refer to the lecture notes when you get stuck, and make up your cheat sheet as you go along.
- Try the Practice Exam on the web site. Under realistic conditions, of course, and mostly to test your cheat sheet.

Small-scale anisotropies in the cosmic microwave background

The satellite observatories WMAP and Planck have both mapped the cosmic background radiation over the whole sky, at angular resolution of a few arcminutes.

- Before these missions, balloon-borne observatories (e.g., MAXIMA, BOOMERANG) had mapped large patches of the sky at similar or higher angular resolution.

The resulting images have resolved the small-amplitude anisotropies in the background radiation.

- The anisotropies are thought to be density-temperature fluctuations due to adiabatic acoustic oscillations, endemic in the Universe before decoupling.
- The WMAP and Planck images represent the fluctuations at the instant decoupling forever stopped the oscillations.
Small-scale anisotropies in the CMB (continued)

Final image of the sky by WMAP, representing nine years of data (Bennett et al. 2013).

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Small-scale anisotropies in the CMB (continued)

Sky image from Planck, based on 30 months of data (Planck Collaboration 2013). Because Planck has higher angular resolution ($\times$) than WMAP, very small anisotropies look brighter to Planck than to WMAP, necessitating the larger temperature scale.

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Small-scale anisotropies in the CMB (continued)

Origin of the small-scale anisotropies:

- Inhomogeneities - peaks and troughs in the density - are to be expected in an expanding gas like that of the Big Bang, even if their contrast is small.
- These inhomogeneities oscillate acoustically - ring like a bell, driving sound waves into their surroundings:
  - Gravity tends to collapse the density peaks, heating them up and increasing the temperature of the radiation (light) therein.
  - This increases the radiation pressure, which pushes it back, then gravity pulls it in again. Repeat.
  - And the opposite happens in the density troughs.
Small-scale anisotropies in the CMB (continued)

This proceeds until decoupling, when the radiation can escape from the matter.

Think of the Universe before decoupling as a resonant cavity (like the pulsating stars we considered earlier): there would be a fundamental mode evident in the spectrum of sound.

Thus a last snapshot of the Universe in the act of this “ringing” is preserved in images of the CMB.

There are lots of resonances, but wavelengths any larger than twice $ct/a$ at decoupling won’t show up in the cosmic background: that is, $\lambda_{\text{max}}/2 = \ell_d \equiv c\theta_d/a_d$.

- $\ell_d$ is called the acoustic horizon; it’s the distance limit for cause and effect, pre-decoupling.

Small-scale anisotropies in the CMB (continued)

The acoustic horizon turns out to be independent of curvature.

For $\Omega < 1$, the relations between $t$ and $a$ for all of the universes considered this semester reduce to the same formula, which we will obtain here for the open matter-dominated universe. (You’ll do another in recitation.)

First note that $\sinh^{-1} x = x - \frac{x^3}{6} + \frac{x^5}{40} + \ldots$, remember that $(1 + x)^n = 1 + nx + \binom{n}{2} x^2 + \ldots$, and expand the terms in our previous solution to third order overall in $x = \sqrt{(1 - \Omega)} a/\Omega \ll 1$:
Small-scale anisotropies in the CMB (continued)

Thus, since \( a \approx 1 \) at decoupling,
\[
f_d = \frac{c_d}{n_d} = \frac{2\Omega_d^{1/2}}{3H_0d_{c}}
\]

no matter what the universe.

The redshift of the decoupling surface, as we’ve seen, is
\[
z_d = 1000 \Rightarrow 1 + z_d = \frac{\Omega_d}{\Omega_{d,0}} = \frac{n_d}{n_{d,0}}.
\]

So the acoustic horizon – the length scale of the fundamental mode of oscillation – is
\[
f_d = \frac{2c}{3H_0d}\Omega(1 + z_d) = 149 \text{ Mpc for } \Omega = \Omega_{d,0} = 0.3.
\]

A new standard ruler, and the curvature of the Universe

A histogram of the small-scale anisotropies as a function of projected linear size (a.k.a. angular power spectrum) will have a strong peak near 149 Mpc – the fundamental – and other peaks for higher-order modes of oscillation.

That is: they comprise a standard ruler, painted on the decoupling surface.

At decoupling the oscillation stops, as the photons get away, but the matter concentrations will still tend to have characteristic size 149 Mpc and this scale should show up in the distribution of galaxies at later times.

Sure enough, it does: these baryon acoustic oscillations are seen at just the right scale in 3-D galaxy distributions at \( z < 0.7 \) (Anderson et al. 2012).
A new standard ruler (continued)

Baryon acoustic oscillations in the power spectrum of the 3-D galaxy distribution at $z = 0.43-0.7$, in the Sloan Digital Sky Survey (Anderson et al. 2012).

The acoustic horizon is a standard ruler and is the same in any universe, but the distance to the decoupling surface, and therefore the angle subtended by a fluctuation, depends on the curvature of the universe. Thus we can measure $K$.

Because it’s a lot of writing, we’ll work out the distance in just one geometry—flat—and merely give answers for the others.

- The distance $\Delta \theta$ is that which light travels from the decoupling surface, which according to the RW interval obeys $ds^2 = 0 = c^2 dt^2 - R^2 dr^2 = c^2 dt^2 - a^2 dr^2$:

$$\Delta r = \int_{r_0}^{r_f} dr = c \int_{t_0}^{t_f} \frac{dt}{a(t)}$$

- We know what $t(a)$ is (lecture, 18 April 2013), but it is quicker to use the chain rule and the Friedmann equation:

$$\Delta r = c \int_{t_0}^{t_f} \frac{dt}{a(t)} = c \frac{1}{H_0} \int_{a_0}^{a_f} \frac{da}{(1 + \Omega) \sqrt{1 + (1 - \Omega) a^2 / \Omega}}$$

where, as usual, $\Omega = \Omega_{m0}$ and $1 - \Omega = \Omega_{\Lambda0}$.

This integral can be done analytically but involves such things as hypergeometric functions of arcsines (see Wolfram Alpha) so I’ll do it numerically. For $\Omega = 0.3$ and $z_d = 1090$,

$$\Delta r = 1.291 \times 10^4 \text{ Mpc}, \quad \theta_d = \frac{r_d}{\Delta r} = 0.66^\circ.$$
Similarly, for a couple of the other universes we’ve considered:

<table>
<thead>
<tr>
<th>Universe</th>
<th>$\Omega$</th>
<th>$\Delta r$ (10^8 Mpc)</th>
<th>$\theta_0$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_M = 0$, $\Omega_{\Lambda} = 0$</td>
<td>-1</td>
<td>2.423</td>
<td>0.83</td>
</tr>
<tr>
<td>$\Omega_M = 0$, $\Omega_{\Lambda} = 0.7$</td>
<td>0</td>
<td>1.291</td>
<td>0.66</td>
</tr>
<tr>
<td>$\Omega_M = 0$, $\Omega_{\Lambda} = 1.0$</td>
<td>+1</td>
<td>0.6979</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Summary: in an angular power spectrum of CMB anisotropies (histogram of fluctuations as a function of angular size), the angle at which the fundamental (largest-angle) peak occurs determines $K$ for the Universe.

Other peaks at smaller angles are harmonics of the oscillation; their amplitudes are sensitive to the $\Omega$s.

Fundamental acoustic mode appears at angular size 0.8°; accounting for details leads to 0.6° for the acoustic horizon (Page et al. 2003).

Thus the Universe appears to be accurately and precisely flat.
A new standard ruler (continued)

Properties of the new flat Universe

The solid curves which run through the data points on the last two pages are fits by a six-parameter $\Lambda$CDM model.

- The six parameters determine the geometry of space through which we view the cosmic microwave background, the acoustic oscillations, and the vicinity of the decoupling surface.
- Many other parameters – notably $H_0$, $\Omega_{\Lambda}$, and the Universe's age – can be derived indirectly from the model but are not used in the fits.
- The experimental uncertainties are small and the fits are good, so the resulting uncertainties in the fitted and derived parameters are very small.

<table>
<thead>
<tr>
<th>Properties of the new flat Universe (continued)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planck collaboration 2013</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$, $\Omega_m$, $H_0$, $\Delta N_{\mathrm{e}}$, $\sigma_8$, $\tau$, $\lambda$, $A$</td>
</tr>
<tr>
<td>Direct collaboration 2023</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$\Omega_{\Lambda}$</td>
</tr>
<tr>
<td>$\Omega_m$</td>
</tr>
<tr>
<td>$H_0$</td>
</tr>
<tr>
<td>$\Delta N_{\mathrm{e}}$</td>
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<tr>
<td>$\sigma_8$</td>
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<tr>
<td>$\tau$</td>
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<tr>
<td>$\lambda$</td>
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<tr>
<td>$A$</td>
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</tbody>
</table>
Disquieting features of the new flat Universe

The cosmological constant itself. This was put into general relativity ad hoc, in Einstein’s time and ours. There was, even originally, no basic-physical motivation, and there is still no consistent explanation of $\Lambda$ in terms of experimentally-identified particles and fields.

- Made more disquieting with the notable lack of theoretical consensus about its nature.
- Made certainly more disquieting by its magnitude: dark energy comprises about 70% of the Universe’s total mass-energy.

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Disquieting features of the new flat Universe (continued)

The Hubble constant. The values of $H_0$, derived from CMB anisotropies are significantly smaller than those from Cepheids and SNe Ia; they disagree at 99% confidence. More ≥

- BAO results for $H_0$ tend to agree with CMB results…
- and geometrically-derived $H_0$ (megamaser-galaxy distances, gravitational lensing, S-Z) tends to agree with the others.
Disquieting features of the new flat Universe (continued)

The distance indicators. Our standard ruler is acoustic oscillation in expanding media with gravitation and radiation in equilibrium. Our standard candle is the SN Ia. These are physically more complex, and less accessible for detailed study, than the tools we used on smaller scales.

- Observation of BAOs certainly will help sort out the physics of the CMB anisotropies.
- But the blast physics in SNe Ia is poorly understood, and the effect on SN Ia yield of low metallicity in the early Universe is unknown.
- Pulsating stars are physically simple by comparison. Look how long it took to get them tamed for use as standard candles...

Anti-Copernicanism. Since mass density decreases with expansion but dark-energy density does not, the "antigravitational" effects of the latter increase with time.

- And we seem to live at a time when the effects of matter and dark energy are close in magnitude. If we had missed, they would have been orders of magnitude different.
- Such fine tuning always arouses the suspicions of scientists, and often (though not always) points toward a conceptual problem or inconsistency.
Disquieting features of the new flat Universe (continued)

Cosmology eventually becomes impossible. Further expansion will be exponential. In a few Hubble times, distant galaxies and the CMB will be redshifted into undetectability. 

So carve your results in stone, save many copies in safe places, and cultivate a reputation for honesty. Your successors will have to take your word for all of this Big Bang stuff.

![Graph showing the scale factor, a(t), over time from present, with different scenarios for \( \Omega_M \) and \( \Omega_\Lambda \).]

- \( \Omega_M = 0.225, \Omega_\Lambda = 0.775 \)
- \( \Omega_M = 0.225, \Omega_\Lambda = 0 \)
- \( \Omega_M = 1, \Omega_\Lambda = 0 \)
- \( \Omega_M = 2, \Omega_\Lambda = 0 \)

\( H_0 = 74.2 \text{ km sec}^{-1} \text{ Mpc}^{-1} \)