Problem 10.1

An electron in the \( n = 2 \) state of hydrogen remains there on average \( \Delta t = 10 \) ns before jumping to the \( n = 1 \) state (the so-called lifetime of the excited state) by emission of light.

(a) Estimate the uncertainty in the energy (\( \Delta E \)) and frequency (\( \Delta \nu \)) of the \( n = 2 \) state.

(b) What fraction of the transition energy, \( \Delta E/(E_2-E_1) \), is this?

(c) What is the wavelength, and width (in nm), of this line in the spectrum of hydrogen?

(d) Show that

\[
\frac{\Delta \lambda \lambda}{\lambda} = \frac{\Delta \nu \nu}{\nu} = \frac{\Delta E}{E_2 - E_1} = 6.46 \times 10^{-9}
\]

Solution 10.1

(a) The minimum uncertainty in the energy is found from Eq. 38-2.

\[
\Delta E \geq \frac{\hbar}{\Delta t} = \frac{\left(1.055 \times 10^{-34} \text{ J} \cdot \text{s}\right)}{\left(1 \times 10^{-8} \text{ s}\right)} = 1.055 \times 10^{-26} \text{ J} = 6.59 \times 10^{-9} \text{ eV} \approx 10^{-7} \text{ eV}
\]

\[
\Delta \nu = \frac{\Delta \omega}{2\pi} = \frac{1}{2\pi} \frac{1}{\Delta t} = \frac{1}{2\pi} \frac{1}{\left(1 \times 10^{-8} \text{ s}\right)} = 16 \text{ MHz}
\]

(b) The transition energy can be found from Eq. 37-14b. \( Z = 1 \) for hydrogen.

\[
E_n = -\left(13.6 \text{ eV}\right) \frac{Z^2}{n^2} \quad \rightarrow \quad E_2 - E_1 = \left[-\left(13.6 \text{ eV}\right) \frac{1^2}{2^2}\right] - \left[-\left(13.6 \text{ eV}\right) \frac{1^2}{1^2}\right] = 10.2 \text{ eV}
\]

\[
\frac{\Delta E}{E_2 - E_1} = \frac{6.59 \times 10^{-8} \text{ eV}}{10.2 \text{ eV}} = 6.46 \times 10^{-9} \approx 10^{-8}
\]

(c) The wavelength is given by Eq. 37-3.

\[
E = \frac{\hbar c}{\lambda} \quad \rightarrow
\]

\[
\lambda = \frac{\hbar c}{E_2 - E_1} = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^8 \text{ m/s}\right)}{\left(10.2 \text{ eV}\right) \left(1.60 \times 10^{-19} \text{ J}/\text{eV}\right)} = 1.22 \times 10^{-7} \text{ m} = 122 \text{ nm} \approx 100 \text{ nm}
\]
Take the derivative of the above relationship to find $\Delta \lambda$.

$$E \equiv E_2 - E_1, \quad \lambda = \frac{hc}{E} \rightarrow d\lambda = -\frac{hc}{E^2} dE \rightarrow \Delta \lambda = -\frac{hc}{E^2} \Delta E = -\frac{\Delta E}{E} \rightarrow$$

$$|\Delta \lambda| = \frac{\Delta E}{E_2 - E_1} = (122 \text{ nm})(6.46 \times 10^{-2}) = 7.88 \times 10^{-7} \text{ nm} = 10^{-6} \text{ nm}$$

(d) Follows from above.

**Problem 10.2**

A free electron has a wave function $\psi(x) = A \sin(2.0 \times 10^8 x)$, where $x$ is given in centimeters. Determine the particle’s (a) wavelength, (b) momentum, (c) speed, and (d) kinetic energy.

**Solution 10.2**

The wave function is given in the form $\psi(x) = A \sin kx$.

(a) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.0 \times 10^8 \text{ m}^{-1}} = 3.142 \times 10^{-10} \text{ m} = 3.1 \times 10^{-10} \text{ m}$

(b) $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{3.142 \times 10^{-10} \text{ m}} = 2.110 \times 10^{-24} \text{ kg} \cdot \text{m/s} = 2.1 \times 10^{-24} \text{ kg} \cdot \text{m/s}$

(c) $v = \frac{p}{m} = \frac{2.110 \times 10^{-24} \text{ kg} \cdot \text{m/s}}{9.11 \times 10^{-31} \text{ kg}} = 2.3 \times 10^6 \text{ m/s}$

(d) $K = \frac{p^2}{2m} = \left(2.110 \times 10^{-24} \text{ kg} \cdot \text{m/s}\right)^2 \frac{1}{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ J/eV})} = 15 \text{ eV}$

**Problem 10.3**

A quantum particle has a wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}} e^{-x/a} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

(a) Find and sketch the probability density.

(b) Find the probability that the particle will be at any point where $x < 0$. 

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(c) Show that $\psi(x)$ is normalized.
(d) Find the probability of finding the particle between $x = 0$ and $x = a$.

**Solution 10.3**

(a) 
\[
|\psi(x)|^2 = \begin{cases} 
\frac{2}{a} e^{-2x/a} & \text{for } x > 0 \\
0 & \text{for } x < 0
\end{cases}
\]

(b) Prob ($x < 0$) = 0

(c) 
\[
\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{0}^{\infty} |\psi(x)|^2 \, dx + \int_{0}^{\infty} |\psi(x)|^2 \, dx = \int_{0}^{\infty} \frac{2}{a} e^{-2x/a} \, dx = 1
\]

(d) Prob ($0 < x < a$) = 0.865

**Problem 10.4**

A single oxygen molecule is confined in a one-dimensional rigid box of width 0.4 cm.
(a) Treating this as a particle in a rigid box, determine the ground-state energy.
(b) If the molecule has an energy equal to the one-dimensional average thermal energy 0.5kT at $T = 300$ K, what is the quantum number $n$?
(c) What is the energy difference between the $n$th state and the next higher state?

**Solution 10.4**

(a) The ground state energy is given by Eq. 38-13 with $n = 1$.

\[
E_1 = \frac{\hbar^2 n^2}{8mL^2} \bigg|_{n=1} = \frac{\left(6.63 \times 10^{-34} \text{ J s}\right)^2}{8(32 \text{ u})(1.66 \times 10^{-27} \text{ kg/amu})(4.0 \times 10^{-3} \text{ m})^2}(1.60 \times 10^{-19} \text{ J/eV})
\]
\[
= 4.041 \times 10^{-19} \text{ eV} \approx 4.0 \times 10^{-19} \text{ eV}
\]

(b) We equate the thermal energy expression to Eq. 38-13 in order to find the quantum number.
\[ \frac{1}{2} kT = \frac{h^2 n^2}{8m} \rightarrow \]
\[ n = 2\sqrt{kTm \frac{I}{\hbar}} = 2\sqrt{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})(32 \text{ u})(1.66 \times 10^{-27} \text{ kg/u}) \left( \frac{4.0 \times 10^{-3} \text{ m}}{6.63 \times 10^{-34} \text{ J s}} \right)} \]
\[ = 1.789 \times 10^8 \approx 2 \times 10^8 \]

(c) Use Eq. 38-13 with a large-\( n \) approximation.

\[ \Delta E = E_{n+1} - E_n = \frac{h^2}{8m} \left[ (n+1)^2 - n^2 \right] = \frac{h^2}{8m} \left( 2n + 1 \right) \approx 2n \frac{h^2}{8m} = 2nE_i \]
\[ = 2 \left( 1.789 \times 10^8 \right) \left( 4.041 \times 10^{-19} \text{ eV} \right) = 1.4 \times 10^{-10} \text{ eV} \]

**Problem 10.5**

An excited H atom is in a 5\( d \) state. (a) Name all the states to which the atom is “allowed” to jump with the emission of a photon. (b) How many different wavelengths are there (ignoring the fine structure)?

**Solution 10.5**

Photon emission means a jump to a lower state, so for the final state, \( n = 1, 2, 3, \) or 4. For a \( d \) subshell, \( l = 2 \), and because \( \Delta l = \pm 1 \), the new value of \( l \) must be 1 or 3.

(a) \( l = 1 \) corresponds to a \( p \) subshell, and \( l = 3 \) corresponds to an \( f \) subshell. Keeping in mind that \( 0 \leq l \leq n - 1 \), we find the following possible destination states: \( 2p, 3p, 4p \), \( 4f \).

(b) In a hydrogen atom, \( l \) has no appreciable effect on energy, and so for energy purposes there are four possible destination states, corresponding to \( n = 2, 3, \) and 4. Thus there are three different photon wavelengths corresponding to three possible changes in energy.

**Problem 10.6**

List the quantum numbers for each electron in the ground state of oxygen \((Z = 8)\).

**Solution 10.6**

For oxygen, \( Z = 8 \). We start with the \( n = 1 \) shell, and list the quantum numbers in the order \((n,l,m_l,m_s)\).
\[
\begin{align*}
(1,0,0,-\frac{1}{2}),&(1,0,0,+\frac{1}{2}),&(2,0,0,-\frac{1}{2}),&(2,0,0,+\frac{1}{2}), \\
(2,1,-1,-\frac{1}{2}),&(2,1,-1,+\frac{1}{2}),&(2,1,0,-\frac{1}{2}),&(2,1,0,+\frac{1}{2})
\end{align*}
\]

Note that, without additional information, there are two other possibilities that could substitute for any of the last four electrons.