Useful equations

Length contraction, time dilation and velocity addition:

\[ \Delta x_1 = \Delta x_2 \sqrt{1 - \frac{V^2}{c^2}} = \Delta x_2 \frac{\gamma}{\gamma} \]

\[ \Delta t_1 = \frac{\Delta t_2}{\sqrt{1 - \frac{V^2}{c^2}}} = \Delta t_2 \gamma \]

\[ v_1 = \frac{v_2 + V}{1 + \frac{v_2 V}{c^2}} \]

where \[ \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \]

Absolute interval = \[ \sqrt{\Delta x_1^2 - c^2 \Delta t_1^2} = \sqrt{\Delta x_2^2 - c^2 \Delta t_2^2} \]

= \[ \sqrt{\Delta x^2 - c^2 \Delta t^2} \]

Useful rearrangements of the Absolute Interval formula:

\[ \Delta t_1 = \frac{1}{c} \sqrt{\Delta x_1^2 - \Delta x_2^2 + c^2 \Delta t_2^2} \]

\[ \Delta x_1 = \sqrt{\Delta x_2^2 - c^2 \Delta t_2^2 + c^2 \Delta t_1^2} \]

Mass-energy equivalence:

\[ E = mc^2 \]

Relativity of mass:

\[ m = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}} = m_0 \gamma \]

Black hole accretion luminosity \( L \), for efficiency \( \varepsilon \) and accretion at rate \( \dot{m} \) (usual units gm/sec or \( M_\odot \)/year):

\[ L = \varepsilon \dot{m} c^2 \quad \text{or} \quad \dot{m} = \frac{L}{\varepsilon c^2} \]

Schwarzschild singularity (event horizon):

\[ C_S = \frac{4\pi GM}{c^2} \]

or \[ M = \frac{c^2 C_S}{4\pi G} \]

Nonrelativistic Doppler effect:

\[ \frac{\lambda - \lambda_0}{\lambda_0} = \frac{V}{c} \]

or \[ \lambda = \lambda_0 \left( 1 + \frac{V}{c} \right) \]

or \[ V = c \left( \frac{\lambda}{\lambda_0} - 1 \right) \]

Evaporation lifetime for a black hole that starts with mass \( M \):

\[ t = \frac{1024 \pi^2 G^2}{h c^4} M^3 \]

\[ = \left( 8.407 \times 10^{-26} \frac{\text{sec}}{\text{gm}^3} \right) M^3 \]

Hubble’s Law:

\[ V = H_0 D \quad \text{or} \quad D = V/H_0 \]

Hubble time:

\[ \frac{1}{H_0} = 1.5 \times 10^{10} \text{ years.} \]

Models of the Universe always predict an age for the Universe that is close to the Hubble time.