This practice exam will do you very little good unless (1) you have mastered the homework and workshop problems, and tried to understand everything discussed in the online solutions for all these problems; (2) you have already composed your one-page Cheat Sheet and want to see if it really has everything you need on it, and nothing you don’t. If you’ve done (1) and (2), feel free to proceed.

If this were a real exam, you would be advised here of the exam rules:

“You may consult only one page of formulas and constants and a calculator while taking this test. You may not consult any books, nor each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly. Exams are due an hour and fifteen minutes after we start, and will be returned to you during the next lecture. Good luck.

“Note:

• “Work first on the problems you find easiest, and come back to harder or less familiar material later. Don’t get stuck.

• “The amount of space left for each problem is not necessarily an indication of the amount of writing it takes to solve it.”

Name: ______________________
Problem 1 (30 points)

Consider two fusion mechanisms: the main proton-proton chain, and deuterium fusion. The masses of protons, deuterons, and helium are

\[
\begin{align*}
m\left(^{1}\text{H}\right) &= 1.6726 \times 10^{-24} \text{ gm}, \\
m\left(^{2}\text{H}\right) &= 3.3436 \times 10^{-24} \text{ gm}, \text{ and} \\
m\left(^{4}\text{He}\right) &= 6.6447 \times 10^{-24} \text{ gm}.
\end{align*}
\]

a. Fill in the blanks in the nuclear-reaction chains for these two processes:

p-p chain I:

\[2^{1}\text{H} \rightarrow ^{2}\text{H} + \underline{\quad} + \underline{\quad} \quad (\times 2)\]

\[^{2}\text{H} + \underline{\quad} \rightarrow ^{3}\text{He} + \underline{\quad} \quad (\times 2)\]

\[2^{3}\text{He} \rightarrow \underline{\quad} + 2^{1}\text{H}\]

Total \[\underline{\quad} \rightarrow \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} \]

d-d fusion:

\[2^{2}\text{H} \rightarrow ^{4}\text{He} + \underline{\quad}\]

b. Calculate the energy released by fusion, and available for the lightweight products (electrons, photons, etc.), in each of these “total” reactions, \(\Delta E_{\text{p-p}}\) and \(\Delta E_{\text{d-d}}\). Express your answer in ergs.
c. Compare the neutrino production by these two mechanisms. If the core of the Sun could plausibly be made of pure deuterium, the fusion of which produced the Sun’s power, would the solar neutrino problem be solved, or does neutrino oscillation still look like a better explanation?
Problem 2 (40 points)

A deuterium star. A $1M_\odot$ main-sequence star like the Sun has a central temperature of $1.57 \times 10^7$ K and the subatomic particles in its center (electrons, protons and other ions) have an average mass of $0.62m_p$. Consider a $1M_\odot$ star made completely of deuterium, $\frac{2}{1}\text{H}$, which produces its energy by d-d fusion into helium. Assume that its internal structure is similar to that of the Sun, apart from its different composition.

a. Calculate the average particle mass in the deuterium star’s interior, assuming that the material is fully ionized. Express your answer in terms of the proton mass $m_p$.

b. Suppose the deuterium star has the same radius as the Sun. Calculate its central temperature.
Problem 2 (continued)

c. Show that, at this temperature, d-d fusion reactions in the deuterium star’s core occur a little less than half as often as p-p reactions in the Sun’s core.
Problem 2 (continued)

d. Using this ratio of fusion rates, and the ratio of energies released in d-d and p-p fusion (Problem 1b), find the luminosity of the deuterium star. Assume that the production of $^4\text{He}$ is limited by the p-p fusions that start the chain, note that two p-p fusions are necessary for one $^4\text{He}$, and use your result from Problem 1b.

e. Calculate the effective temperature of the deuterium star.
Problem 3 (20 points)

You have measured the $U$, $B$, and $V$ magnitudes of many stars in an open cluster. You plot $U - B$ against $B - V$, and compare this graph to that for nearby main-sequence stars with no extinction. Here’s what you get:

Estimate the extinction at visible wavelengths between us and the open cluster.
Problem 4 (30 points)

You have observed an eclipsing double-line spectroscopic binary, in which the two sets of lines are identical, but for which the Doppler shifts are sinusoidal and opposite. The velocity amplitudes are both \( v = 43 \text{ km sec}^{-1} \), and the period \( P = 31 \text{ days} \). The visual magnitude of the binary is \( V = 10 \).

a. Make some reasonable assumptions and deductions about the shape and orientation of the orbits, and the relative masses of the stars in the binary. Then calculate the separation of the stars, expressing your answer in Solar radii.

b. Calculate the masses of the stars in the binary, expressing your answer in solar masses.
Problem 4 (continued)

c. Calculate the visual magnitude of each star.
Problem 5 (20 points)

A $1M_\odot$ star is observed to oscillate in brightness with a period of 48 hours and an amplitude of 1 visual magnitude.

a. Assume it is uniform in density. What is the radius of the star, in Solar radii?

b. In what phase of evolution is the star likely to be?
Problem 6 (30 points)

Short answers.

a. Estimate the dates, during the next year, for which the sidereal time at midnight (standard time) is $0^h, 6^h, 12^h, \text{ and } 18^h$.

b. A celestial object is above the horizon for 12 hours. What is its declination?
Problem 6 (continued)

c. Explain why there is a maximum degeneracy pressure that electrons are capable of exerting, and therefore a maximum mass for white dwarf stars.

d. Describe the difference in nature between the pulsations exhibited by the Sun – the “five minute oscillations” – and those found in Mira, Cepheid and RR Lyrae variables.
Problem 6 (continued)

e. The brightest stars in a certain constellation are identified in the following star chart: there are five second-magnitude stars and one with apparent magnitude zero. Larger symbols are used to denote brighter stars.

Identify the constellation from these choices:

- Cassiopeia.
- Centaurus.
- Cassiopeia, as seen from $\alpha$ Centauri.
- Centaurus, as seen from $\alpha$ Cassiopeiae.

f. Suppose I know how to find the Winter Triangle: the nearly-equilateral figure at whose vertices are Betelgeuse, Sirius and Procyon. Describe to me how to locate the constellations Monoceros and Orion, relative to the Winter Triangle.