Astronomy 203 Problem Set #8

Due 10 December 1999

1. A pressure-scanned Fabry-Perot interferometer. At normal atmospheric pressure and room temperature, the index of refraction of CO₂ is 1.0045. At the same temperature and a pressure of 4 atmospheres, the index is 1.0180. For constant temperature, the index varies linearly with pressure between these extremes. Using a few complete sentences, suggest a way of using this effect to tune a Fabry-Perot interferometer. For a pressure varying between 1-4 atmospheres, and an interferometer with spacing 0.3 cm and finesse \( Q = 30 \), operating at a wavelength 0.55\( \mu \)m, what range of wavelengths is covered by the scan? Which Fabry-Perot order is used? How many FWHM resolution elements are contained in the scan? (Many high-resolution, visible-wavelength Fabry-Perot spectrometers employ this principle.)

2. Beam size and spectral resolution of a Fabry-Perot. An incoherent detector looks through a Fabry-Perot interferometer at normal incidence, with a beam of small angular radius \( \theta \).

   a. Show that the detector is therefore sensitive to a range of wavelengths, varying from \( \lambda = 2nd/m \) to \( \lambda' = 2nd(1 - \theta^2/2)/m \), and that a wavelength resolution element can therefore be no smaller than

   \[
   \Delta\lambda = \frac{\lambda \theta^2}{2}.
   \]

   We will refer to this result as the beam-divergence limit to the spectral resolution of a Fabry-Perot interferometer.

   b. Suppose you wanted to have the beam be 0.1 radian (5.7°) in radius. For a Fabry-Perot with a finesse of 20, what is the highest order number you can use before the beam-divergence limitation on the spectral resolution is equal to the reflectance-limited resolution?

   c. Suppose further that you really need better spectral resolution than that. Suggest an optical configuration for the Fabry-Perot that will overcome the beam-divergence limitation.

3. Diffraction grating measurements of the sodium D-lines (\( \lambda = 0.58959, 0.58900 \)\( \mu \)m).

   a. A diffraction grating has \( 10^4 \) rulings uniformly spaced over 2.5 cm. It is illuminated by yellow light from a low-pressure sodium-vapor lamp, at normal incidence. At what angles will the first-order maxima occur for these lines?

   b. How many rulings must a diffraction grating have in order barely to resolve them in third order?

   c. In a particular grating the D-lines are viewed in third order at 80° to the normal and are barely resolved. How far apart are the grating rulings?

4. Grating spectrometer design. Taking the limits of the visible spectrum to be \( \lambda = 0.43 - 0.68 \)\( \mu \)m, design a grating that will spread the first order spectrum through an angular range of 20°. Use any incidence angle you like. Report the incidence angle, the range of diffracted angles, and the proper blaze angle.

5. The entrance slit of a grating spectrometer. Consider a telescope with a grating spectrometer, as shown schematically in [Figure 1](#). The telescope has diameter \( D \) and focal length \( F \), the grating-spectrometer
collimating mirror has diameter $d$ and focal length $f$, and the light can enter the spectrometer through a slit of width $x$. The spectrometer is in Littrow mode ($\theta_m = -\theta_i$).

![Figure 1: optical setup for Problem 5.](image)

**a.** If $x$ is not zero, the “collimated” light hitting the grating has some angular spread. Explain why, and show that the angular spread is given by $\Delta \theta = \frac{x}{f}$, and thus that the resolution of the spectrometer is

$$\frac{\Delta \lambda}{\lambda} = \frac{1}{\lambda} \frac{2 a \cos \theta_m}{m}$$

—that is, you can make the resolution better by making $x$ smaller.

**b.** Of course, diffraction prevents you from making $x$ arbitrarily small. Show that the smallest $x$ is allowed to be is $1.2 \frac{f \lambda}{d}$, and therefore that the smallest $\Delta \lambda / \lambda$ can be is

$$\frac{\Delta \lambda}{\lambda} = \frac{2.4}{mN}$$

**c.** Suppose you want the slit to match the image size from the telescope's beam, an angle $\phi$ in size. Find an expression for the resolution, in terms of $\phi$, $D$, and $d$. (This, and the previous result, show how one “matches” a grating spectrometer to a telescope.)