Today in Physics 217: dielectrics

- Finish things from last lecture
- Dielectrics
- Electric polarization and bound charge
- Calculation of field and potential from uniformly-polarized objects

Dielectric materials

Solids are generally composed of neutral atoms and molecules, some of which have built-in, permanent dipole moments and some of which are simply polarizable. For non-conducting solids,

- there is zero dipole moment on large scales, since the orientation of permanent dipoles is generally random.
- immersion in an electric field polarizes atoms and molecules, and tends to align their permanent dipole moments.
- this polarization is characterized by a dipole moment per unit volume, in the same direction as the applied field. Non-conducting solids that can be polarized in this way are called dielectrics.

Electric polarization and bound charge

The electric polarization $P$ is a vector quantity:

$$ P = \frac{d}{dt} p = \int P dt' $$

The potential from a lump of polarized matter is

$$ V = \frac{1}{\epsilon} \cdot P = \int \frac{\epsilon}{\epsilon'} \cdot d\tau' $$

Now,

$$ \frac{\epsilon}{\epsilon'} = -\nabla \cdot \left( \frac{1}{\epsilon} \right) = -\nabla \left( \frac{1}{\epsilon} \right) $$

as we saw long time ago (lecture, 18 September)
Electric polarization and bound charge (continued)

(Reminder: take the Cartesian components of \( \mathbf{r} = \mathbf{r}' \) to be \( X,Y,Z \), those of \( \mathbf{r} \) to be \( x,y,z \), those of \( \mathbf{r}' \) to be \( x',y',z' \). Then

\[
V = \left[ \begin{array}{c} \frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial z} \\
\frac{\partial}{\partial x'} \\
\frac{\partial}{\partial y'} \\
\frac{\partial}{\partial z'} \\
\end{array} \right] = \nabla_s \quad V = \left[ \begin{array}{c} \frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial z} \\
\frac{\partial}{\partial x'} \\
\frac{\partial}{\partial y'} \\
\frac{\partial}{\partial z'} \\
\end{array} \right] = -\nabla_s \quad \text{So}
\]

\[
V = \int P \cdot \nabla' \left( \frac{1}{\epsilon} \right) \, dt' 
\]

Electric polarization and bound charge (continued)

Let's integrate this by parts, using product rule #5:

\[
V (\mathbf{A}) = \int \nabla \cdot \mathbf{A} + \mathbf{A} \cdot (\nabla) 
\]

\[
V = \int P \cdot \nabla' \left( \frac{1}{\epsilon} \right) \, dt' = \int \nabla' \left( \frac{P}{\epsilon} \right) \, dt' - \int \frac{1}{\epsilon} \nabla' \cdot P \, dt' 
\]

\[
= \oint_S P \cdot \mathbf{n} \, dt - \int_V \frac{1}{\epsilon} \nabla' \cdot P \, dt' . 
\]

Define the surface and volume bound charge densities:

\[
\sigma_b = P \cdot \mathbf{n} \quad (\mathbf{n} = \text{outward normal of } S) 
\]

\[
p_b = -\nabla \cdot P 
\]

Then the potential takes on a familiar form:

\[
V = \oint_S \sigma_b \cdot \mathbf{n} \, dt - \int_V p_b \, dt' . 
\]

Bound charge is the charge displaced by the field into dipolar form. Note that for a uniform (constant) polarization, the bound volume charge density is zero:

\[
p_b = -\nabla \cdot P = 0 
\]

leaving just surface bound charge, like so:

Neutralized

(c) University of Rochester
Calculation of field and potential from uniformly-polarized objects

You’ll be pleased to know that you’ve already made major progress toward calculations involving uniformly-polarized objects, because you can make them by superposition of uniformly-charged objects. Consider a uniformly polarized cylinder:

Same as two uniform, oppositely charged cylinders, displaced infinitesimally along the axis (see problem 2.27, HW #4).

Calculation of field and potential from uniformly-polarized objects (continued)

Or a uniformly-polarized sphere, which would work out the same as problem 2.18 (HW #3).

Calculation of field and potential from uniformly-polarized objects (continued)

We found in that problem that the field in the overlap region (or, if you like, inside the polarized sphere) is

Outside, the field is just that of a simple dipole, since both spheres act like point charges located at their centers:

Note that the (in principle infinitesimal) displacement distance $d$ drops out of the problem when everything is expressed in terms of polarization.