Today in Physics 217: simple active AC circuits

- Operational amplifiers (opamps)
- Amplification with opamps
- Arithmetic with opamps
- Integration and differentiation with opamps
- Analog computers

Circuit diagram for the venerable 741 opamp (National Semiconductor Corp.).
Passive and active circuit elements

The circuit elements we’ve seen so far – capacitors, resistors and inductors – are called passive. These devices always obey the same relation between their current and voltage, independent of how large or small the current or voltage is.

That is, they have the same impedance at all voltages.

Active electronic devices are those whose current-voltage properties (impedance) change with voltage.

Semiconductor diodes or vacuum tubes, for instance. Some active devices can amplify: they allow one to control and change a large electrical power by changing a small electrical power, or vice versa.

Examples: transistors, vacuum-tube triodes, and circuits made from these devices.
Operational amplifiers

This is not a course in electronics; we will not discuss the details of transistors. But one particular class of transistor circuit that can provide amplification is simple enough to learn the basics of in a few minutes: the operational amplifier, or opamp.

- Internally, they’re not simple, and the finest details of their performance is way beyond the scope of our discussion, but most of what they do can be described in only two concise statements.

Let’s learn enough about opamps to use them in simple active AC circuits, thereby enriching substantially the number of AC-circuit situations in which you will be able to solve problems.
Opamps (continued)

Single opamp in eight-pin dual-inline package (DIP).

Connection diagram. +V and -V are DC power supplies, usually +15 and -15 volts.

Circuit-diagram symbol: leave off DC power-supply and offset-null connections.
Opamps (continued)

An opamp has two inputs and one output.

- An increase in the potential at the noninverting input leads to an increase in the potential at the output.
- An increase in the potential at the inverting input leads to a decrease in the potential at the output.
- Thus the output change is proportional to the difference between the input voltage change.
- The proportionality constant (the open-loop gain, or amplification) is huge, usually larger than 100000.
Opamps (continued)

The two rules of opamps:

- If a current path is provided between the output and the inverting input, the output voltage will adjust automatically to produce zero voltage difference between the two inputs.
- The inputs draw no current.

Connection of an amplifier’s output to its inverting input is called negative feedback.
Amplification with opamps

The two rules make it easy to find the relation between input and output voltages, DC or AC. Here are a few examples involving simply amplification of a single voltage. (Here, and throughout, voltage means potential relative to ground.)

- Inverting amplifier:
  - voltage at + is zero, so the voltage at – is too (rule #1). Thus $I = V_{\text{in}}/R_1$
  - flows in $R_1$. But the same current must flow in $R_2$ (rule #2), so

$$V_{\text{out}} = -IR_2 = -\frac{R_2}{R_1} V_{\text{in}}.$$
Amplification with opamps

Noninverting amplifier. Here the voltage at – must be \( V_{\text{in}} \) (rule #1), and the wire connecting – to the point between the resistors carries no current (rule #2), so that point is also at voltage \( V_{\text{in}} \):

\[
I = \frac{V_{\text{in}}}{R_1},
\]

\[
V_{\text{out}} = I (R_1 + R_2) = \frac{V_{\text{in}}}{R_1} (R_1 + R_2) = \left( 1 + \frac{R_2}{R_1} \right) V_{\text{in}}.
\]
Arithmetic with opamps

- Summing amplifier:

\[ V_- = 0 \quad (\text{rule #1}) \]
\[ \Rightarrow I_1 = \frac{V_1}{R}, \quad I_2 = \frac{V_2}{R}. \]
\[ I = I_1 + I_2 \quad (\text{rule #2}) \]
\[ \Rightarrow V_{out} = -IR = -(V_1 + V_2). \]

Easily extended to as many inputs as one wants.
Arithmetic with opamps (continued)

- Differential amplifier:

\[
I_2 = \frac{V_2}{2R} \quad \text{(rule #2)}
\]

\[
V_+ = I_2 R = \frac{V_2}{2} = V_- \quad \text{(rule #1)}
\]

\[
I_1 = \frac{1}{R} (V_1 - V_-)
\]

\[
= \frac{1}{R} \left( V_1 - \frac{V_2}{2} \right)
\]

\[
V_{out} = V_- - I_1 R
\]

\[
= \frac{V_2}{2} - \left( V_1 - \frac{V_2}{2} \right) = V_2 - V_1.
\]
Calculus with opamps

So far, these circuits would operate equally well on constant or time-varying voltages. Here we begin purely AC cases.

- **Differentiator:**

\[
I = \frac{dq}{dt} = C \frac{dV_{\text{in}}}{dt} \quad \text{(rule #2)},
\]

\[
= -\frac{V_{\text{out}}}{R} \quad \text{(rule #1)}.
\]

\[
\Rightarrow V_{\text{out}} = -RC \frac{dV_{\text{in}}}{dt}.
\]
Calculus with opamps (continued)

- Integrator:

\[ I = \frac{V_{\text{in}}}{R} \quad \text{(rule #1)}, \]

\[ = \frac{dq}{dt} = -C \frac{dV_{\text{out}}}{dt} \quad \text{(rule #2)}. \]

\[ V_{\text{out}} = -\frac{1}{RC} \int V_{\text{in}}(t)dt. \quad V_{\text{in}} \]

The integration would go on forever. If one doesn’t want it to – for instance, if one wanted to change functions at the input – one closes the reset switch briefly. A new integration begins as soon as it’s opened.
Calculus with opamps (continued)

One can make differentiators and integrators with inductors, too, but in general it works better with capacitors; inductors can’t be manufactured quite as precisely as capacitors, and don’t behave as close to ideally as capacitors do.

\[ V_{\text{out}} = -\frac{L}{R} \frac{dV_{\text{in}}}{dt} \]

\[ V_{\text{out}} = -\frac{L}{R} \int V_{\text{in}} \, dt \]
Analog computers

Since we have circuit modules that integrate and differentiate voltages, and circuit modules that add and subtract voltages, we can now “write” linear differential equations as circuits. Take, for example, the differential equation we used on Wednesday:

$$\frac{d^2 q}{dt^2} + \frac{\omega_0}{Q} \frac{dq}{dt} + \omega_0^2 q = 0.$$

We can generate terms that look like these with two integrators:

\[
(RC)^2 \frac{d^2 V}{dt^2} - RC \frac{dV}{dt} - V.
\]
Analog computers (continued)

Let’s take the linear and first-derivative terms and add them together:

\[(RC)^2 \frac{d^2 V}{dt^2}\]

To set these equal, simply connect them with a wire!
Analog computers (continued)

Thus

\[
\frac{d^2 V}{dt^2} + \frac{R'}{R^2 C} \frac{dV}{dt} + \frac{1}{(RC)^2} V = 0.
\]
Analog computers (continued)

So to simulate
\[ \frac{d^2 q}{dt^2} + \frac{\omega_0}{Q} \frac{dq}{dt} + \omega_0^2 q = 0, \]
we just need to choose the resistors and capacitors so that
\[ \frac{R'}{R^2C} = \frac{\omega_0}{Q} \quad \text{and} \quad \frac{1}{(RC)^2} = \omega_0^2, \]
and the numbers we’d measure for \( V(t) \), by simply hooking a voltmeter or oscilloscope to the output of the second integrator, would be proportional to \( q(t) \).

Our circuit is therefore an analog computer that solves the damped-harmonic-oscillator equation of motion.
Analog computers (continued)

Now, this doesn’t seem too interesting; we already know the solution to this equation. But suppose we needed to know the solution to a differential equation of the form

$$\frac{d^7 V}{dt^7} + \alpha \frac{d^4 V}{dt^4} + \beta \frac{dV}{dt} + \gamma V = V_0(t).$$

How long would it take you to solve this analytically? How long would it take you to write a computer program to solve it digitally? A long time in either case.

How long would it take to build the corresponding analog computer and measure the answer? About half an hour. You’d make it out of seven integrators, two inverting amplifiers, and an adder.
Analog computers (continued)

Because analog computers are so fast at solving differential equations in this manner, they have been important in modelling and simulation of physical situations.

- And analog computers using multipliers as well as adders can even simulate the solutions of nonlinear differential equations, such as those that crop up in fluid mechanics. This class of differential equations is much more difficult to solve analytically than linear DEs.

- Unfortunately one can’t make a multiplier simply with opamps, so we can’t address this interesting problem here.
Recommended reading

...about opamps and simple opamp circuits:

...about analog computers: