If this were a real exam, you would be reminded here of the **exam rules**: “You may consult *only* one page of formulas and constants and a calculator while taking this test. You may *not* consult any books, nor each other. All of your work must be written on the attached pages, using the reverse sides if necessary. The final answers, and any formulas you use or derive, must be indicated clearly. Exams are due three hours after we start, and will be available to be reclaimed on Monday. Good luck.”

This practice test is meant mostly to illustrate the style, length, variety, and degree of difficulty of the problems on the real exam; not all the topics we have covered are represented here. This is not to say that these topics won’t appear on the real exam, or that there will be problems just like the ones here on the real exam! You should expect problems on any topic we have covered in lecture, recitation and on the homework.

For best results please try hard to work out the problems before you look at the solutions.

Name: ______________________
Problem 1 (30 points)

a. An infinite cylinder with radius $2R$ is charged uniformly, with charge density $\rho$, except for an infinite cylindrical hole parallel to the cylinder's axis. The hole has radius $R$ and is tangent to the exterior of the cylinder. A short chunk of the cylinder is shown in the accompanying figure.

Calculate the electric field everywhere inside the hole, and sketch the lines of $E$ on the figure.
b. An infinite cylindrical wire with radius $2R$ carries a uniform current density $J$, except inside an infinite cylindrical hole parallel to the wire's axis. The hole has radius $R$ and is tangent to the exterior of the wire. A short chunk of the wire is shown in the accompanying figure.

Calculate the magnetic field everywhere inside the hole, and sketch the lines of $B$ on the figure.
Problem 1 (continued)

c. An infinite cylindrical “flux tube” with radius 2R carries a uniform magnetic field $B$, parallel to the cylinder's axis, except inside an infinite cylindrical hole parallel to the flux tube’s axis. The hole has radius $R$ and is tangent to the exterior of the flux tube. A short chunk of the flux tube is shown in the accompanying figure. The magnetic field is zero inside the hole and outside the flux tube, but where it exists, it is increasing linearly with time:

$$B(t) = B_0 \frac{t}{t_0}.$$ 

Calculate the electric field everywhere inside the hole, and sketch the lines of $E$ on the figure.
Problem 2 (30 points)

a. Calculate the electric field at a point on the axis of a uniformly-charged circular disk, a distance $z$ from the circle's center. The disk has surface charge density $\sigma$ and radius $a$. 
Problem 2 (continued)

b. The charged circular disk from part a is set into rotation about its axis, with angular frequency $\omega$. Calculate the magnetic field $B$ at a point on the axis a distance $z$ from the center of the disk.
Problem 3 (20 points)

Consider the reference point for electric potential to be at infinity for both parts of this problem.

a. A conducting sphere with radius $a$ is in an infinite vacuum. What is its capacitance?

b. The same sphere is placed in an infinite, weakly-conducting medium with resistivity $\rho$. What is the resistance between the sphere and infinity?
Problem 4 (10 points)

a. Calculate the magnetic field inside a long solenoid with radius $a$, $N_a$ turns per unit length, and current $I$.

b. Inside the long solenoid and parallel to it, there is a short solenoid with radius $b$, $N_b$ turns per unit length, and length $\ell$. What is the mutual inductance of the two solenoids?
Problem 5 (20 points)

An infinite, charged, straight wire (charge per unit length $\lambda$) lies parallel to the $z$ axis, and parallel to two infinite, grounded conducting planes which intersect at a $90^\circ$ angle. The charged wire lies a distance $a$ from each plane. Calculate the electric field $E$ at point $A$, $2a$ from each plane on the same side as the line charge, and at point $B$, $a/2$ from each plane but on the side opposite the line charge.
Problem 6 (20 points)

A sphere with radius $R$ carries a polarization $P = Kr$, where $K$ is a constant and $r$ is the vector distance from the center.

a. Find the surface and volume bound charges.

b. Calculate the fields $E$ and $D$ everywhere.
Problem 7 (20 points)

a. A circular wafer is made of very weakly conducting material with resistivity $\rho$ and dielectric constant $\varepsilon$. Its radius is $r$ and its thickness is $\ell \ll r$. Highly conductive, metallic electrodes cover the circular faces.

Under the assumption that the material’s conductivity doesn't affect the capacitance, calculate the resistance and capacitance of the wafer with its electrodes.

b. Suppose that the wafer is charged up with a battery with voltage $V$, and that the battery is disconnected at $t = 0$. Write one differential equation describing the charge on the electrodes, and the similar equations describing the potential difference $V(t)$, and the electric field $E(t)$ within the wafer.

(Hint: The wafer's resistance and capacitance have the same potential difference, and can therefore be represented by two elements in parallel.)
Problem 7 (continued)

c. Solve the differential equation to obtain the electric field between the electrodes as a function of time.

d. In terms of the resistivity and dielectric constant of the material, what is the time constant? Comment on the generality of this result, considering the dependence of the time constant on the shape of the wafer.