Highlights of the New Material for Midterm #4

CAVEAT EMPTOR
I strongly advise that you do not take this to be a substitute for your own gathering of the material in your mind because that won’t work. Rather, I suggest that you look over these topics and make sure you’ve thought about and understood these before the exam.

Also, please note that I am only attempting to summarize the most important points in the material. Nowhere am I implying that you need only know these items. You are still responsible for all that is covered in classes, workshops and assigned reading and problems.
New Material

- Magnetic Materials
- Faraday’s Law
- Mutual and Self Inductance
- LR Circuits
- AC Circuits

- TA Review Before Next Midterm
  - December 1st, 8:30pm, B&L 109
Magnetic Materials Review

- In response to an external magnetic field, materials develop internal magnetization

\[ B_{\text{total}} = B_0 + \mu_0 M = (1 + \chi_m) B_0 = \kappa_m B_0 \]

- **Paramagnetism**
  - weak alignment of \( M \) and \( B \), \( 0 < \chi_m << 1 \)

- **Diamagnetism**
  - weak anti-alignment of \( M \) and \( B \), \( -1 << \chi_m < 0 \)

- **Ferromagnetism**
  - strong alignment of \( M \) and \( B \), \( \chi_m >> 1 \)

These are very strong effects, magnetization dominates field!
It’s the Floating Frog!

• A diamagnetic frog levitates in a strong magnetic field as shown at right
• If gravity causes a downwards force in the picture at right, where is the magnitude of the field greatest? At the bottom of the page.
• Better be able to explain why… this brings together a lot of key ideas.

\[ U = -\vec{\mu} \cdot \vec{B}(z) = |\chi_m| \left| \vec{B}(z) \right|^2 \]

\[ F_z = -\frac{dU}{dz} = -(-\chi_m)2B \frac{dB}{dz} \]

\[ \therefore F_z > 0 \text{ if } \frac{dB}{dz} < 0 \]

If potential energy is highest at bottom of page, then the frog feels an upward force.

Therefore, \( B \) is largest at bottom of the page.
The Big Picture
which you should be able to explain...

• **Electrostatics**  
  • motion of “q” in external E-field  
  • E-field generated by $\Sigma q_i$

• **Magnetostatics**  
  • motion of “q” and “I” in external B-field  
  • B-field generated by “I”

• **Electrodynamics**  
  • time dependent B-field generates E-field  
    • ac circuits, inductors, transformers, etc  
  • our last topic (not this exam)  
    time dependent E-field generates B-field  
    • electromagnetic radiation - *light*
Faraday’s and Lenz’s Laws

- a changing magnetic flux through a loop induces a current in that loop

\[ \varepsilon = -\frac{d\Phi_B}{dt} \]

negative sign indicates that the induced EMF opposes the change in flux

- Need to also understand Faraday’s Law in terms of Electric Field

- It does need a loop of wire to be true! This field can accelerate free charges not in a wire!
Applications

• should be able to explain how a generator works

\[ E \equiv -\frac{d\Phi_B}{dt} = -\frac{d}{dt}[BA\cos\omega t] = BA\omega \sin\omega t \]

• should be able to explain how a transformer works

• should be able to explain our eddy current demonstrations done in class
Big Picture of Inductance

• A coil produces a magnetic field

• That magnetic field produces magnetic flux in that coil and adjacent coils

• If current changes in Coil 1, flux changes in Coils 1 and 2

• That change of flux causes an EMF which induces current!
  – in Coil 2 “Mutual Inductance”;
  – in Coil 1, “Self Inductance”
**Inductors**

- Current in Coil 1 causes
  - Magnetic flux in Coils 1 & 2
- Changing current…
  - Induces EMF in Coils 1 & 2
  - Self and Mutual Inductance
- Depends on geometry only!
  - Unit is *Henry*, $T \cdot m^2/A \equiv \Omega \cdot s$

- Mutual Inductance
  
  \[
  M = \frac{\Phi_B \text{ through } a \text{ from } b}{I_b} = \frac{\Phi_B \text{ through } b \text{ from } a}{I_a}
  \]

- Self-Inductance
  
  \[
  L \equiv \frac{\Phi_B}{I}
  \]
  
  - Self-Inductance for Solenoid
    
    \[
    L = \mu_0 \frac{N^2}{l} \pi r^2
    \]
  - $L$ increases with Ferromagnetic core
    
    $\mu_0 \rightarrow \mu = \kappa_M \mu_0$
Summary so far...

Want to find voltage given a current, or find current if given a voltage.

\[ V = \frac{Q}{C} = \int I \, dt \]

Voltage determined by integral of current and capacitance

\[ V = IR \]

Voltage determined by current itself and resistance

\[ V = L \frac{dI}{dt} = L \frac{d^2 Q}{dt^2} \]

Voltage determined by derivative of current and inductance
Rules of Thumb for Inductors in Circuits

- After circuit has had a long time to settle…
  - What is $dl/dt$? Zero
  - So the EMF across the inductor is? Zero
  - So it acts like a wire (no potential difference)

- When something changes in the circuit
  - How much should I be allowed to change instantaneously? Not a lick!
  - What is the mechanism for opposing change? Provide an EMF!
  - How much EMF? $IR$
  - Change is exponential in time, $\tau = L/R$
Energy in the *Electric* and *Magnetic* Fields

Energy stored in a capacitor ...

\[ U = \frac{1}{2} C V^2 \]

... energy density ...

Energy stored in an inductor ....

\[ U = \frac{1}{2} LI^2 \]

... energy density ...

\[ u_{\text{electric}} = \frac{1}{2} \varepsilon_0 E^2 \]

\[ u_{\text{magnetic}} = \frac{1}{2} \frac{B^2}{\mu_0} \]
LC Oscillations

\[ L \frac{d^2 Q}{dt^2} + \frac{Q}{C} = 0 \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]
Response to an AC Voltage

- **R**: \[ V_R = R I_R = \varepsilon_m \sin \omega t \quad \Rightarrow \quad I_R = \frac{\varepsilon_m}{R} \sin \omega t \]
  - \( V \) in phase with \( I \)

- **C**: \[ V_C = \frac{Q}{C} = \varepsilon_m \sin \omega t \quad \Rightarrow \quad I_C = \omega C \varepsilon_m \sin(\omega t + 90^\circ) \]

- **L**: \[ V_L = L \frac{dI_L}{dt} = \varepsilon_m \sin \omega t \quad \Rightarrow \quad I_L = \frac{\varepsilon_m}{\omega L} \sin(\omega t - 90^\circ) \]
  - \( V \) lags \( I \) by 90°

- Voltage/Current relationship across a single circuit element can be divided into:
  - magnitude
    \[ I = \frac{\varepsilon_m}{"X"} \sin(\omega t - \phi) \]
    \[ X_L \equiv \omega L \quad X_R \equiv R \]
  - relative phase
    » leading, lagging

\[ X_C \equiv \frac{1}{\omega C} \]
Impedance Networks

\[ Z_R = R = Re^{i0} \]

\[ Z_c = \frac{1}{i\omega C} = -i = e^{-i\pi/2} \]

\[ Z_L = i\omega L = \omega L e^{i\pi/2} \]

- Combining impedances in series and parallel is just as simple as it was with resistors
  - But here impedances are complex numbers!

\[ V_0 e^{i\omega t} \sim Z_1 \quad \frac{1}{Z_p} = \frac{1}{Z_1} + \frac{1}{Z_2} \]

\[ V_0 e^{i\omega t} \sim Z_1 \quad Z_s = Z_1 + Z_2 \]
Resonance in LRC Series Circuit

\[ V_{\text{max}} = I_{\text{max}} |Z_{eq}| \]

- So when does the current reach a maximum if the voltage and \( R, L, C \) are fixed?

\[ |Z_{eq}| = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2} \]

- The current \( I_{\text{max}} \) will be a maximum at the resonant frequency \( \omega_0 \) which makes the impedance \( Z \) purely real (\( R \) only)!

i.e.: when \( \omega_0 L - \frac{1}{\omega_0 C} = 0 \) or \( \omega_0 = \frac{1}{\sqrt{LC}} \)
Power in LRC Circuit

• The power supplied by the emf in a series LRC circuit depends on the frequency $\omega$ (maximum power is supplied at the resonant frequency $\omega_0$).

• Can calculate from either power supplied by generator or power dissipated in resistor

$$P(t) = \varepsilon(t)I(t) = (\varepsilon_m \sin \omega t)(I_m \sin(\omega t - \phi))$$

• average power delivered in a cycle.

$$\langle P(t) \rangle = \varepsilon_m I_m \langle \sin \omega t \sin(\omega t - \phi) \rangle$$

$$\varepsilon_{rms} = \frac{1}{\sqrt{2}} \varepsilon_m \quad I_{rms} = \frac{1}{\sqrt{2}} I_m \quad \Rightarrow \quad \langle P(t) \rangle = \varepsilon_{rms} I_{rms} \cos \phi$$

• so power delivered also depends on relative phase of voltage and current in the generator