Exam 1 (February 11, 2010)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show your work where indicated.

Problem 1 (7 pts):

The sketch below shows four situations in which charged particles are fixed in place on an axis. In which situation(s) is there a point to the left of the particles where an electron will be in equilibrium?

Problem 2 (7 pts):

The sketch below shows three situations involving a charged particle and a uniformly charged spherical shell. The charges are given, and the radii of the shells are indicated. With numerals “1”, “2” and “3”, rank the situations according to the magnitude of the force on the particle due to the presence of the shell, greatest first.

Problem 3 (6 pts):

A closed three-dimensional surface is so constructed that, at all points on the surface, the electric field vector points inward. Therefore, it is true that

a) the surface encloses a net positive charge.

b) the surface encloses a net negative charge.

c) the surface encloses no electric charge.

d) the area vector, dA, for each segment of the surface is necessarily parallel to the electric field vector.

e) the area vector, dA, for each segment of the surface is necessarily perpendicular to the electric field vector.
**Problem 4 (5 pts):**

A square has equal positive charges at three of its corners, as shown in the sketch below. Circle the vector that best shows the direction of the electric field at point P.

**Problem 5 (15 pts, show work):**

A conducting ball of mass 0.2 kg is thrown straight upward from the surface of the earth with a speed of 2 m/s. The ball carries an electric charge of 1 Coulomb and thrown upward in a big room where there is a constant electric field of 0.5 N/C upward. To what height above the surface of the earth does the ball rise?

From 1st kinematics equations for constant acceleration:

\[ a = \frac{qE - mg}{m} = \frac{(1)(0.5) - (2)(9.8)}{0.2} \]

\[ V_y^2 = V_{oy}^2 + 2a(y - y_0) \]

At top: \[ V_y = 0 \]

\[ V_{oy}^2 = 2ah \]

\[ h = 0.3 \text{ m} \]

**Problem 6 (15 pts, show work):**

An electric field is \( \vec{E} = +400 \text{ N/C \hat{i}} \) for \( x > 0 \) and \( \vec{E} = -400 \text{ N/C \hat{i}} \) for \( x < 0 \). A cylinder of length 0.3 m and radius 0.1 m has its center at the origin and its axis of symmetry along the x axis such that one end of the cylinder is at \( x = +0.15 \text{ m} \) and the other is at \( x = -0.15 \text{ m} \). What is the net charge inside the cylinder? (Remember that the “\( \hat{i} \)-hat” vector is a unit vector along the x axis.)

- a) Zero.
- b) \( 1.1 \times 10^{-10} \text{ C} \)
- c) \( 2.2 \times 10^{-10} \text{ C} \)
- d) \( 4.5 \times 10^{-10} \text{ C} \)
- e) \( 2.2 \times 10^{-9} \text{ C} \)
- f) \( 4.5 \times 10^{-9} \text{ C} \)

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \]

\[ \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \]

\[ \int \vec{E} \cdot d\vec{A} = EA + EA \]

\[ Q_{enc} = \epsilon_0 \frac{21 \pi \times 0.1^2}{\epsilon_0} = (8.85 \times 10^{-12}) (2)(400)(0.1)^2 (3.14) \]
Problem 7 (15 pts, show work):

"Why, yes, Virginia. Physics DOES help us understand some aspects of sex." … Reproduction in many flower species depends on insects carrying pollen grains from one flower to another. One way honeybees do this is by collecting the grains electrically, because the bees are usually positively charged. When a bee hovers near a flower’s anther, which is electrically insulated, the pollen grains (which are moderately conducting) jump to the bee, where they cling during the flight to the next flower.

Give plausible and brief explanations of (a) why the pollen grains jump to the bee and (b) why it is that the bee is typically electrically charged in the first place using the concepts we have discussed recently in class.

Assuming the bee is electrically charged, the presence of the bee will induce a charged dipole on the pollen grain with the opposite sign charge on the side of the pollen closest to the bee. Because the opposite sign charge is closest to the bee (relative to the same sign charge part of the dipole) the pollen will be attracted to the bee by the Coulomb force.

The charge on the bee comes about due to the rapid moving movement through the air. The rubbing of different materials together often causes a transfer of charge due to the fact that different materials have different degrees of attraction for atomic electrons. (Different electronegativities)

Problem 8 (15 pts, show work):

A plane with a uniform (constant) area charge density, \( \sigma \), passes through a region containing a sphere of radius R and with charge of +Q distributed along its surface. The plane passes through the sphere of charge such that the distance of closest approach of a point on the plane to the center of the sphere is R/2. Determine the electric field at point P (which is at the center of the sphere of charge). (Hint: If this takes you more than a few minutes you should move on and come back to this. It is likely you need to rethink your strategy.)
Problem 9 (15 pts, show work):

Consider a non-conducting spherical shell with inner radius A and outer radius B. The inside cavity of the shell ($r<A$) is empty. The shell carries a charge density, $\rho = br^2$, where $b$ is a constant (with appropriate units) in the region $A<r<B$. The total amount of charge on the spherical shell is $+Q$. Determine the electric field for the regions $r<A$, $A<r<B$, and $B<r$.

For $r<A$
choose spherical gaussian

Surface when $r<A$

$sE \cdot dA = \frac{Q \text{ enc.}}{\varepsilon_0}$

$4\pi r^2 E_1 = 0$

$E_1 \big|_{r<A} = 0$

For $A<r<B$
choose spherical gaussian surf.

$sE \cdot dA = \frac{Q \text{ enc.}}{\varepsilon_0}$

$1E_1 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$E_1 \big|_{A<r<B} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$

For $r>B$
choose gaussian surf.

$sE \cdot dA = \frac{Q \text{ enc.}}{\varepsilon_0}$

$1E_1 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$E_1 \big|_{r>B} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$

$E$ is radially outward due to spherical symmetry of the problem.

$E_1 \big|_{A<r<B} = \frac{b}{6\varepsilon_0} \left( \frac{r^4 - A^6}{r^2} \right) = \frac{b}{6\varepsilon_0} r^2$
Exam I Formulas

\[ \vec{F} = q \vec{E} \]
\[ \vec{F} = \frac{k q_1 q_2}{r_{12}^2} \hat{r}_{12} = \frac{1}{4 \pi \varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \]
\[ \phi_E = \oint \vec{E} \cdot d\vec{A} \]
\[ \oint \vec{E} \cdot d\vec{A} = \frac{\text{Q enclosed}}{\varepsilon_0} \]
\[ \vec{E} = \int \frac{k dQ \hat{r}}{r^2} \]

\[ \int u^m du = \frac{u^{m+1}}{m+1} \]
\[ \int \frac{du}{u} = \ln |u| \]
\[ \int e^u du = e^u \]
\[ \int \frac{xdx}{(x^2 + a^2)^{1/2}} = \frac{x}{2} \ln(x^2 + a^2) \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/N\text{m}^2 \]
\[ k = 8.99 \times 10^9 \text{ N} \text{m}^2/\text{C}^2 \]
\[ q = 1.6 \times 10^{-19} \text{ coulombs} \]

\[ \sin \theta = \frac{b}{h} \quad \cos \theta = \frac{a}{h} \quad \tan \theta = \frac{b}{a} \]

Sphere: \[ A = 4\pi r^2 \quad V = \frac{4}{3} \pi r^3 \]
Cylinder: \[ A = 2\pi r L + 2\pi r^2 \quad V = \pi r^2 L \]

\[ V = V_0 + at \]
\[ x = x_0 + V_0 t + \frac{1}{2} at^2 \]
\[ a_c = \frac{mV^2}{r} \]
\[ S = r \theta \]
\[ KE = \frac{1}{2} mV^2 \]
\[ PE_{spring} = \frac{1}{2} kx^2 \]